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Relationship Between Multi-agent Logic Systems W and Weak S5 Systems

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Abstract

W, which was proposed in [22] [26], is a multi-agent logic system based on shared common knowledge views. In this paper, we investigate the relationship between W and the classical weak S5 multi-agent logic systems, S4 and KD4. Suppose q is a multi-agent formula, T is a theory, neither of them contains a fool modal operator, then the main conclusions are: 1. If $T \vdash_W q$ then $T \vdash_{KD4} q$. 2. If $T \vdash_{KD4} q$ then $K0T \vdash_W K0q$. 3. If $T \vdash_{S4} q$ then $K0T^0_{Ag1} \vdash_W K0q^0_{Ag1}$. Suppose T is a theory, p is a formula which may contain a fool reasoner, then: 4. If $T \vdash_W q$ then $T^1_0 \cup ... \cup T^n_0 \vdash_{S4} q^1_0 \vee ... \vee q^n_0$. When W has only one normal agent, that is $Ag = \{0, 1\}$, then for every formula q which contains only a normal agent, we have: 5. $\vdash_{S4} q$ iff $\vdash_W q^1_0$

1 Introduction

W is a multi-agent logic system based on shared common knowledge view. Details about its introduction can be found in [22] [26]. Main opinions about W are as follows:

1. Tautology is known by every agent.

   Every tautology is decidable by every agent. For example, let i be an agent, p is a statement, then, $Ki(p \vee \neg p)$ is true and agent i can prove that $p \vee \neg p$ is true.

2. The knowledge known by a normal agent i can be inconsistent with the real world. This means that, $(Kip) \wedge \neg p$ is consistent in W [Ref [26], Example 3.1]. So the knowledge axiom K, $Kip \rightarrow p$ in modal logic S5 can not be held in W.

3. Real world knowledge may not be known by any agent. This means that the necessitation rule in modal logic S4 and KD4 is not included in W.

   One of the basic ideas behind logic system W is: a true real world knowledge 'p' is not necessarily obvious to every agent. In other words, real world knowledge is not shared common knowledge.

4. Every agent has positive introspective ability (So W has the 4-axiom), but no negative introspective ability (Hence W does not have the 5-axiom).

5. Considering the real world knowledge and agent's knowledge, we assume that if agent i knows $\neg p$ then agent i should not know p. That is, if an outsider (or a

god) observes that agent i knows $\neg p$, then the outsider will assume that agent i does not know p. This traditional D-axiom implies that axiom $\neg Kifalse$ is also valid in W, so no agent in W can believe a False Statement.

6. Common knowledge is the typical knowledge of W. Common knowledge is black board knowledge. In most precious published papers such as: [1][7][6][5], it was assumed that common knowledge should be defined by infinite deductions. Our opinion about common knowledge is that it should have the infinite deductive properties, but it should not be limited to and defined by those properties.

   Common knowledge has the following properties in W:

   (a) First, as in [15], a fool reasoner, denoted by 0, is introduced in W, and what the fool knows is common knowledge. If K0p appears in a theory, p should be common knowledge.

   (b) Tautology is common knowledge.

   (c) Common knowledge is true in the real world. This means that W contains the axiom $K0p \rightarrow p$ and the safeness rule $K0p \Rightarrow p$.

   Further, the fact that common knowledge should be true in the real world, is also common knowledge. This means W takes $K0(K0p \rightarrow p)$ as its axiom.

   (d) If p is common knowledge, then for every agent i, Kip is also common knowledge.

   (e) Compared with the common knowledge definition in [1], one of the most important aspects of W is that W only takes care of how to use common knowledge rather than concerning with what common knowledge is.

W, being a multi-agent logic system based on shared common knowledge view, is complete [26]. In this paper we investigate the deeper relationship results between W and traditional multi-agent weak S5 systems (S4 and KD4).

The paper is organized as follows. In section 2, we briefly introduce the logic system W, its semantics and main properties. In section 3, we briefly introduce the traditional weak S5 multi-agent logic system, S4 and KD4. We describe their Kripke Possible semantics and give their complete results. In section 4, we investigate the relationship between W and KD4 system. The main conclusion is: Suppose T is a theory, q is a formula, both of them contain no fool modal operator, then:

If $T \vdash_W q$ then $T \vdash_{KD4} q$.

If $T \vdash_{KD4} q$ then $K0T \vdash_W K0q$

This shows that although W is a multi-agent system based on shared common knowledge, it is no weaker then KD4.

In section 5, we study the relationship between W and S4 systems. We conclude that for every theory T and formula q which may contain a fool modal operator,

if $T \vdash_W q$ then $T_0^1 \cup ..., T_0^n \vdash_{S4} q_0^1 \vee ... \vee q_0^n$.

Here S_0^i is the result of S's substitution of a fool modal operator by Ki.

Inversely, suppose theory T and formula q contain no fool modal operator, then we have:

If $T \vdash_{S4} q$ then $K0T_{Ag1}^0 \vdash_W K0q_{Ag1}^0$.

An interesting conclusion is, if W contains only one fool reasoner and one normal agent, then we have:

$\vdash_{S4} q$ iff $\vdash_W q_1^0$,

So W will be reduced to a S4 system when it contains only one normal agent.

It should be noticed that in this paper, we only consider the propositional multi-agent logic system. For first order multi-agent systems, we have similar results.

2 Logic W and its Main Properties

Suppose At is a set of primitive statements. $Ag = \{0, 1, ..., n\}$ is a set of agents, in which 0 is called the fool agent, the rest are called the normal agent or agent if it is not confused. Informally, 0's knowledge is common knowledge, which is known by all agents.

First, we define the syntax of the well-founded formulas based on At and Ag.

Definition 2.1 A well-founded formula based on At and Ag can be inductively defined as follows:

1. If $p \in At$, then p is a well-founded formula.

2. If p, q are well-founded formulas, $i \in Ag$, then Kip, $(\neg p), (p \rightarrow q)$ are also well-founded formulas.

3. All well-founded formulas are defined by the finite compositions of steps 1 and 2. □

We denote the set of all the well-founded formulas based on At and Ag, by L.

We use special symbols to abbreviate some formulas. We write $(p \vee q)$ for $(\neg p \rightarrow q)$, $p \wedge q$ for $\neg(p \rightarrow \neg q)$, $p \equiv p$ for $(p \rightarrow q) \wedge (q \rightarrow p)$. Assume formula P to be a basic formula if P contains no modal operator.

The axioms and inference rules of W are defined as shown below.

Definition 2.2 W's axioms:

A1. $K0p$, if p is any tautology.

A2. $K0(K0p \rightarrow K0Kip)$.

A3. $K0(Ki(p \rightarrow q) \rightarrow (Kip \rightarrow Kiq))$.

A4. $K0(K0p \rightarrow p)$.

A5. $K0(Kip \rightarrow KiKip)$.

A6. $K0(Ki\neg p \rightarrow \neg Kip)$.

W's inference rules are

Modus Ponens: $p, p \rightarrow q \Longrightarrow q$

Safeness rule: $K0p \Longrightarrow p$ □

Suppose T is a theory. As in [2], we can define the prove relationship between T and well-formed formula p. We denote this by $T \vdash_W p$, where p is called the consequence of T. Obviously, the consequence set of T is $Cons_W(T)$. That is, $Cons_W(T) = \{p | T \vdash_W p\}$.

Theorem 2.1 (Deduction Theorem, Ref [26] Theorem 2.4) .

Suppose T is a theory, p, q are two formulas, then $T \bigcup \{p\} \vdash_W q$ *if and only if* $T \vdash_W p \rightarrow q$. □

Corollary 2.2 Suppose T is a theory, q is a formula, if $T \not\vdash_W q$ then $T \cup \{q\}$ is consistent. □

Theorem 2.3 *For every formula q,* $\vdash_W q$ *iff* $\vdash_W K0q$ □

Corollary 2.4 For every formula q and every agent $i \in Ag$, if $\vdash_W q$ then $\vdash_W Kiq$ □

Theorem 2.5 (Ref [26] Theorem 2.7) .

Suppose T is a theory and $T = Cons_W(T)$. *For any agent* $i \in Ag$, *let* $T/Ki = \{p | Kip \in T\}$, *then* $T/Ki = Cons_W(T/Ki)$. □

Corollary 2.6 [Ref [26] Corollary 2.8].

Suppose p1, ..., pn, q are well-formed formulas, and $i_1, ..., i_k$ are agents. If $p1, ..., pn \vdash_W q$, then $K_{i_1}...K_{i_k}p1, ..., K_{i_1}...K_{i_k}pn \vdash_W K_{i_1}...K_{i_k}q$. □

The reverse dpes not hold. For example, $K0p \vdash_W K0Kip$, but $p \not\vdash_W Kip$.

Definition 2.3 [W-Kripke Structure, Ref [26] Definition 3.1].

Suppose L is a language based on At and Ag. $\kappa = (W, \pi, w0, R0, R1, ..., Rn)$ is a Kripke structure based on L, where W is a non-empty set, called the world set. $w0 \in W$ is called an initial world; π is a map from W to the subset of At; R0,R1,...,Rn are relations on W. Say structure κ is a W-Kripke structure, if κ satisfies the following four conditions:

1. Every Ri $(i = 0, 1, ..., n)$ is transitive;
2. For every $i = 1, ..., n, Ri \subseteq R0$;
3. R0 is reflexive;
4. Every Ri is serial;

That is, for every world $w \in W$, every agent $i \in Ag$, the set $\{w'|(w, w') \in Ri\}$ is not empty. □

Generally, we denote an id for the reflexive relation on W, $id = \{(w, w)|w \in W\}$.

Definition 2.4 Suppose $\kappa = (W, \pi, w0, R0, R1, ..., Rn)$ is a W-Kripke structure. For every $w \in W$, we define the semantics entailment relation $\kappa, w \models_W q$, as follows:

1. If $p \in At$, then $\kappa, w \models_W p$ iff $p \in \pi(w)$
2. $\kappa, w \models_W \neg p$ iff $\kappa, w \not\models_W p$
3. $\kappa, w \models_W p \rightarrow q$ iff if $\kappa, w \not\models_W p$ or $\kappa, w \models_W q$
4. For every $i \in Ag$, $\kappa, w \models_W Kip$ iff for every $w' \in W$, if $(w, w') \in Ri$, then $\kappa, w' \models_W p$ □

Definition 2.5 Suppose $\kappa = (W, \pi, w0, R0, R1, ..., Rn)$ is a W-Kripke structure,

Say formula p is valid in W-Kripke structure κ, denoted by $\kappa \models_W p$, if $\kappa, w0 \models_W p$;

Say theory T is valid in W-Kripke structure κ, denoted by $\kappa \models_W p$, if, for every formula $p \in T$, p is valid in κ;

Say formula p is a semantic entailment of theory T, denoted by $T \models_W p$, if for every W-Kripke structure κ, if T is valid in κ, then p is also valid in κ;

We denote the set of all the semantic entailment of theory T by $Th_W(T)$. □

Definition 2.6 Suppose T is a theory, say T is complete if $T = Cons_W(T)$ and for every formula $p \in L$, we have $p \in T$ or $\neg p \in T$. □

Theorem 2.7 (Complete Theorem, Ref [26] Theorem 5.7) .

1. *Formula p is consistent iff p is satisfiable.*
2. *For every consistency theory T, $Th_W(T) = Cons_W(T)$.* □

3 Multi-agent Weak S5 Systems

In this section, we give a brief introduction about traditional knowledge-based multi-agent logic systems, weak S5 systems such as S4, KD4. Notice that there is no fool reasoner in traditional multi-agent logic systems. So we only discuss the language L1 based only on normal agent set $Ag1 = Ag - \{0\} = \{1, ..., n\}$ and propositional set At. Obviously, L1 is a subclass of the language L which we discussed in section 2.

Definition 3.1 [The Knowledge-based Multi-agent Logic system S4].

For every L1's formulas p, q, agent $i \in Ag1$.

S4's Axioms:

AS1: p, if p is a tautology.

AS2: $Kip \rightarrow p$,

AS3: $Ki(p \rightarrow q) \rightarrow (Kip \rightarrow Kiq)$

AS4: $Kip \rightarrow KiKip$

AS5: $Ki\neg p \rightarrow \neg Kip$

S4's Inference Rules:

Modus Ponens: p, $p \rightarrow q \Rightarrow q$

Necessitation Rule: $p \Rightarrow Kip$. □

Generally AS2 is called knowledge axiom or T-axiom, AS3 is called K-axiom, or distributed axiom, AS4 is called positive introspective axiom or 4-axiom, AS5 is called D-axiom.

KD4 is the S4 logic system deleting the axioms AS2.

Definition 3.2 Say Kripke-structure $\kappa =< W, \pi, R1, ..., Rn >$ is a S4-Kripke structure, if

1. $W \neq \{\}$, W is called a possible world set.
2. π is a map from W to the 2^{At}.
3. R1, ..., Rn are relations on W such that every Ri is reflective, transitive and serial.

When Ri in S4-Kripke structure is not required reflective, then the Kripke structure is called KD4-Kripke structure. □

Suppose $X = S4$ or $KD4$, $\kappa =< W, \pi, R1, ..., Rn >$ is a X-Kripke structure. For every $w \in W$, every formula p, we define $\kappa, w \models_X p$ as follows:

Definition 3.3 $\kappa, w \models_X p$

For every formula $q \in L1$, every world $w \in W$, we define $\kappa, w \models_X q$ as follows:

1. $\kappa, w \models_X q$ iff $q \in \pi(w)$, if $q \in At$.

2. $\kappa, w \models_X \neg q$ iff $\kappa, w \not\models_X q$.

3. $\kappa, w \models_X p \rightarrow q$ iff $\kappa, w \not\models_X p$ or $\kappa, w \models_X q$.

4. For every $i \in Ag1$, $\kappa, w \models_X Kip$ iff for every $w' \in W$, if $(w, w') \in Ri$ then $\kappa, w' \models_X p$ □

Definition 3.4 Suppose $\kappa =< W, \pi, R1, ..., Rn >$ is a X-Kripke structure, T is a theory, p is a formula

Say p is valid in κ, denoted by $\kappa \models_X p$, if for every world $w \in W$, $\kappa, w \models_X p$.

Say T is valid in κ, denoted by $\kappa \models_X T$, if for every formula $p \in T$, $\kappa \models_X p$.

Say p is a X-entailment consequence of T, if for every X-Kripke structure κ, if T is valid in κ, then p is also valid in κ.

We denote the set of all the X-entailment consequences of T as $Th_X(T)$. □

Theorem 3.1 *For every theory S and formula q in L1, we have [Ref [2]]*

1.$S \models_{S4} q$ *iff* $S \vdash_{S4} q$

2.$S \models_{KD4} q$ *iff* $S \vdash_{KD4} q$ □

In the following section, we will discuss the relationship between W and X.

Briefly, the main differences between W and X are:

1. S4, KD4 has Necessitation Inference Rule: $p \Rightarrow Kip$. But W has not this inference rule, W has only Safeness Rule $K0p \Rightarrow p$.

2. W has a fool reasoner, but S4, KD4 have not.

3. Every axiom of W can be viewed as the fool's common knowledge, but axioms of S4, KD4 can only be viewed as agent's knowledge.

4. In W, the knowledge axiom only holds for fool reasoner, that is only $K0p \rightarrow p$ hold in W. In S4, the knowledge axiom holds for every agent. Of course in KD4, no agent has a knowledge axiom.

5. W has a good computational property, the deduction properties [Ref Theorem 2.1], but S4, KD4 do not have this property. For example, $p \vdash_{S4} Kip$, but $\not\vdash_{S4} (p \rightarrow Kip)$.

4 Relationship between KD4 and W

Theorem 4.1 *For every theory T and formula q of L1, if* $S \models_W q$ *then* $S \models_{KD4} q$.

Proof:

Suppose the above statement is not true. That is there are some theory S and formula q on L1 such that $S \models_W q$ *but* $S \not\models_{KD4} q$.

Since $S \not\models_{KD4} q$, *there must be a KD4-Kripke-Structure* $\kappa =< W, \pi, R1, ..., Rn >$ *such that* $\kappa \models_{KD4} S$ *but* $\kappa \not\models_{KD4} q$. *So there must be a world* $w' \in W$ *such that* $\kappa, w' \models_{KD4} \neg q$.

Now, we construct a W-Kripke-Structure as following:

$\kappa_1 =< W_1, w0, \pi_1, R0_1, R1_1, ..., Rn_1 >$ *such that:*

1. $W_1 = W$, $w0 = w'$, $\pi_1 = \pi$, *For i=1,...,n,* $Ri_1 = Ri$.

2. $R0_1 = trans(id \cup R1 \cup ..., \cup Rn)$ [1]

It is easy to check that κ_1 *is a W-Kripke-Structure.*

Inductively on formula's length, we can prove that:

For every formula q in L1, every world $w \in W$, $\kappa, w \models_{KD4} q$ **iff** $\kappa_1, w \models_W q$.

If q is an atom, then it is obviously true.

Suppose the above statement is true for the formulas whose length is not greater than t.

If q is $\neg p$, *and p's length is not greater than t, then for every world* $w \in W$, $\kappa, w \models_{KD4} q$ *iff* $\kappa, w \not\models_{KD4} p$ *iff* $w \in W$, $\kappa_1, w \not\models_W p$ *iff* $\kappa_1, w \models_W q$.

If q is $p1 \rightarrow p2$, *p1, p2's length is not greater than t, then it is also easy to prove the above statement.*

If q is Kip, then For every world $w \in W$, $\kappa, w \models_{KD4} q$ *iff for every* $(w, w') \in Ri$, $\kappa, w' \models_{KD4} p$ *iff for every world* $(w, w') \in Ri$, $\kappa_1, w' \models_W p$ *iff* $\kappa_1, w \models_W q$.

So we have inductively proved our main statement.

Since $\kappa \models_{KD4} S$, $\kappa, w' \models_{KD4} \neg q$, *so we have* $\kappa \models_W S$ *and* $\kappa \not\models_W q$.

From $S \models_W q$ *and the assumption* $\kappa \models_W S$, *we get* $\kappa \models_W q$.

It is obviously a contradiction.

And we prove our theorem. □

Corollary 4.2 For every theory S and formula q on L1, if $S \vdash_W q$ then $S \vdash_{KD4} q$. □

[1] trans(S) is the least transitive relationship which contain the relationship S

Theorem 4.3 *For every theory S and formula q on L1, if we have* $S \models_{KD4} q$ *then* $K0S \models_W K0q$.

Proof:

Suppose there are theory S and formulas q on L1 such that $S \models_{KD4} q$ *and* $K0S \not\models_W K0q$.

Obviously we can find a W-Kripke structure $\kappa =< W, \pi, w0, R0, R1, ..., Rn >$ *such that*

$\kappa, w0 \models_W K0S$, *but* $\kappa, w0 \not\models_W K0q$

Now we construct a KD4-Kripke-Structure $\kappa_1 =< W_1, \pi_1, R1_1, ..., Rn_1 >$ *such that* $\kappa_1 \models_{KD4} S \cup \neg q$.

KD4-Kripke-Structure $\kappa_1 =< W_1, \pi_1, R1_1, ..., Rn_1 >$ *is constructed as follows:*

1. $W_1 = \{w | (w0, w) \in R0\}$.

Obviously we can see that:

For every $w \in W_1$, $\kappa, w \models_W S$.

There is a $w' \in W_1$, $\kappa, w' \not\models_W q$.

2. For every $w \in W_1, \pi_1(w) = \pi(w)$

3. For every $i = 1, ..., n$, $Ri_1 = Ri \cap (W_1 \times W_1)$.

It is easy to prove (by inductive on formula's length) that:

For every formula $p \in L1$ **and** $w \in W_1$, $\kappa, w \models_W p$ **iff** $\kappa_1, w \models_{KD4} p$.

Since for every $w \in W_1$, $\kappa, w \models_W S$ *and* $\kappa, w' \not\models_W q$, *we have* $\kappa_1 \models_{KD4} S$, *and* $\kappa_1, w' \not\models_{KD4} q$. *This is contradiction to* $S \models_{KD4} q$. *And hence we prove our theorem.* □

Corollary 4.4 For every theory S, formula q of L1, if $S \vdash_W q$ then $K0S \vdash_{KD4} K0q$. □

5 Relationship Between W and S4

Since KD4 is a sub-logical system of S4, from corollary 4.2 we have:

Corollary 5.1 For every theory S, formula q of L1, if $S \vdash_W q$ then $S \vdash_{S4} q$ □

This means that W's inference ability about normal agents is not greater then S4. But in which sense does it reach the inference ability of S4? In the following we will discuss this problem.

Definition 5.1 Formula Translation.

Suppose P is a formula in L, $< i1, ..., ik >$ is a agent sequence in which every two elements are different. $< j1, ..., jk >$ is another agent sequence. We say $P^{<j1,...,jk>}_{<i1,...,ik>}$ is P's translation through substituting K_{il} by K_{jl} (here $l = 1, ..., k$).

1. If P contains no modal operator, then $P^{<j1,...,jk>}_{<i1,...,ik>} = P$.
2. $(\neg P1)^{<j1,...,jk>}_{<i1,...,ik>} = \neg P1^{<j1,...,jk>}_{<i1,...,ik>}$.
3. $(P1 \to P2)^{<j1,...,jk>}_{<i1,...,ik>} = (P1^{<j1,...,jk>}_{<i1,...,ik>}) \to (P2^{<j1,...,jk>}_{<i1,...,ik>})$.
4. $(KiP1)^{<j1,...,jk>}_{<i1,...,ik>} =$
   if $i = im, m = 1, ..., k$ then $Kjm(P1^{<j1,...,jk>}_{<i1,...,ik>})$ else $Ki(P1^{<j1,...,jk>}_{<i1,...,ik>})$. □

We denote $P^{<0,...,0>}_{<1,...,n>}$ by P^0_{Ag1}, $P^{<i,...,i>}_{<0,1,...,n>}$ by P^i_{Ag} and $P^{<j>}_{<i>}$ by P^j_i. Suppose S is a theory of L, we denote $\{P^i_{Ag1} | P \in S\}$ by S^i_{Ag1}, $\{P^i_{Ag} | P \in S\}$ by S^i_{Ag} and $\{P^j_i | P \in S\}$ by S^j_i.

Theorem 5.2 *For every theory S and formula q of L, we have*

If $S \vdash_W q$ then for every $i \in Ag1$, $S^1_0 \cup ... \cup S^n_0 \vdash_{S4} q^1_0 \vee ... \vee q^n_0$.

Proof:

We can inductively prove this theorem on q's proof length in W.

When the length is 0.

If $q \in S$, since for $i = 1, ..., n$, $q^i_0 \in S^i_0$, $S^1_0 \cup ... \cup S^n_0 \vdash_{S4} q^1_0 \vee ... \vee q^n_0$ obviously hold.

If q is W's axioms in the form of $K0p$ (p is a tautology), $K0(K0p \to K0Kip)$, $K0(K0p \to p)$, $K0(Kip \to KiKip)$, $K0(Ki(p1 \to p1) \to (Kip1 \to Kip2))$, $K0(Ki\neg p \to \neg Kip)$, then it is not difficulty to prove that $q^1_0 \vee ... \vee q^n_0$ is S4's conclusion. So the theorem also holds.

Now suppose the above statement is true when the proof length is not greater then t.

Suppose q's proof length in W is (t+1). There are three cases to get q.

One case is that $q \in S$ or q is W's axioms. It is obviously true in this case.

The second case is that there is a formula p, such that p and $p \to q$'s proof length is not greater then t.

According to the induction step, $S^1_0 \cup ... \cup S^n_0 \vdash_{S4} (p \to q)^1_0 \vee ... \vee (p \to q)^n_0$ and $S^1_0 \cup ... \cup S^n_0 \vdash_{S4} p^1_0 \vee ... \vee p^n_0$. Since for every $i = 1, ..., n$, $(p \to q)^i_0$ is $(p^i_0 \to q^i_0)$, it is easy to prove that $S^1_0 \cup ... \cup S^n_0 \vdash_{S4} q^1_0 \vee ... \vee q^n_0$

The third case is that formula q is obtained from $K0q$ and $K0q$'s proof length is not greater than t.

According to the deduction step, $S^1_0 \cup ... \cup S^n_0 \vdash_{S4} (K0q)^1_0 \vee ... \vee (K0q)^n_0$, Since for $i = 1, ..., n$, $(K0q)^i_0$ is $Ki(q^i_0)$ and $Ki(q^i_0) \vdash_{S4} q^i_0$, we get $S^1_0 \cup ... \cup S^n_0 \vdash_{S4} q^1_0 \vee ... \vee q^n_0$.

Thus concludes our proof. □

When theory S and formula q are all L1's, we can also get Corollary 5.1 from theorem 5.2. Above results show that W's inference is safe in S4.

On the other side, it is easy to prove:

Theorem 5.3 *For every theory S and formula q of L1, we have*
if $S \vdash_{S4} q$ *then* $K0S^0_{Ag1} \vdash_W K0q^0_{Ag1}$. □

Proof:

We prove the above theorem inductively on q's proof length in S4.

When q's proof length in S4 is 1.

1. $q \in S$, it is obvious that $K0S^0_{Ag1} \vdash_W K0q^0_{Ag1}$.

2. $q \in Axioms(S4)$, then $K0q^0_i$ must be the conclusion of W. So $K0S^0_{Ag1} \vdash_W K0q^0_{Ag1}$.

Suppose above statement is true when q's length is not greater then t.

Now let q's proof length be t+1. Then there must be the following cases to infer q.

1. $q \in S$ or $q \in Axioms(S4)$.

In this case, we can see that the theorem is true.

2. q is obtained from $p \rightarrow q$ and p. Then by induction, $p \rightarrow q$ and p's proof length are not greater than t. So we have

$K0S^0_{Ag1} \vdash_W K0p^0_{Ag1}$ and $K0S^0_{Ag1} \vdash_W K0(p^0_{Ag1} \rightarrow q^0_{Ag1})$.

So we get $K0S^0_{Ag1} \vdash_W K0q^0_{Ag1}$.

3. q is Kip whose proof length is not greater than t.

We have $K0S^0_{Ag1} \vdash_W K0p^0_{Ag1}$. Since $K0p^0_{Ag1} \vdash_W K0K0p^0_{Ag1}$, we get $K0S^0_{Ag1} \vdash_W K0q^0_{Ag1}$.

Hence we have inductively proved this theorem.

Corollary 5.4 For every formula q, $q \in L1$, if $\vdash_{S4} q$ then $\vdash_W q^0_{Ag1}$. □

Suppose $Ag = \{0, 1\}$, then from corollary 5.1 and corollary 5.4, we can get:

Theorem 5.5 *For every theory S, formula q on L1, we have*
$S \vdash_{S4} q$ *iff* $S^0_1 \vdash_W q^0_1$ □

6 Conclusion

In recent years, the representation and reasoning of knowledge and common knowledge in a distributed multi agent system have become a more and more important research topic in the AI field. Most of the formal research is based on the traditional modal logic tools. Common knowledge is a much more difficulty concept to formalize. According to McCarthy, Konolige and other's idea, we adapted a fool reasoner in multi-agent logic systems, and established a complete logic system W. In this paper, we pay our attention to the relationship between W and the traditional multi-agent logic system, weak S5 systems including S4 and KD4. Important results have been achieved. They are:

Suppose T is a theory, q is a formula on L1, then:

1. If $T \vdash_W q$ then $T \vdash_{KD4} q$.
2. If $T \vdash_{KD4} q$ then $K0T \vdash_W K0q$
3. if $T \vdash_{S4} q$ then $K0T^0_{Ag1} \vdash_W K0q^0_{Ag1}$.

Suppose T is a theory, q is a formula on L, then:

4. if $T \vdash_W q$ then $T^1_0 \cup ... \cup T^n_0 \vdash_{S4} q^1_0 \vee ... \vee q^n_0$.

Suppose $Ag1 = \{1\}$, q is a formula on L1, we have:

5. $\vdash_{S4} q$ iff $\vdash_W q^0_1$

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