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# Bottom-Up Abduction by Model Generation

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## Abstract

We investigate two realizations of parallel abductive reasoning systems using the model generation theorem prover MGTP. The first one, called the MGTP+MGTP method, is a co-operative problem-solving architecture in which model generation and consistency checking communicate with each other. There, parallelism is exploited by checking consistencies in parallel. However, since this system consists of two different components, the possibilities for parallelization are limited. In contrast, the other method, called the Skip method, does not separate the inference engine from consistency checking, but realizes both functions in only one MGTP that is used as a generate-and-test mechanism so that consistency checks are automatically performed in reasoning processes. In this method, multiple models can be kept in distributed memories, thus a great amount of parallelism can be obtained. We also attempt the upside-down meta-interpretation approach for abduction, in which top-down reasoning is simulated by a bottom-up reasoner. Evaluations of these abductive systems that are applied to design problems are also described in this paper.

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## 1 Introduction

*Abduction*, an inference to explanation, has recently been recognized as a very important form of reasoning for various AI problems that deal with commonsense knowledge as well as scientific and engineering knowledge. For example, in diagnosis, plan recognition and design, when we observe the behavior of a system, we want to identify the hypotheses that can explain the observation. Also, in natural language understanding, sophisticated user interfaces and communication among intelligent agents, it is recognized that an explanatory capability may play a crucial role [Charniak and McDermott, 1985]. One of the most popular formalizations of abduction in AI defines an *explanation* as a set of hypotheses that, if combined with the background theory, logically entails the given observed formula. This *deductive-nomological* view of abduction, of which Peirce's theory of abduction was the forerunner, has enabled abduction to be implemented using deduction, in particular with resolution-based proof procedures. Along this line, there are a number of proposals for resolution-based abductive systems [Pople, 1973; Cox and Pietrzykowski, 1986; Poole *et al.*, 1987; Stickel, 1989; Inoue, 1991; Demolombe and Fariñas del Cerro, 1991].

Thus, we can expect that studies on automated abduction may fill the gap between traditional, fast deductive techniques and more advanced, AI-oriented commonsense reasoning. From the viewpoint of automated deduction and theorem proving, however, automated abduction is a hard and challenging problem. This is because:

1. Abduction is not a proof finding (or refutation) problem but a *consequence finding* problem (see [Inoue, 1991; Inoue, 1992]).
2. Usually, each abductive explanation is required to be *consistent* with the background theory. While consistency checking is expensive (undecidable in general), it is essential for some practical applications of abduction (design problems, for example), since we are interested in systems that can reject inconsistent theories to obtain acceptable theories.

In this paper, we propose several techniques for implementing abduction that use fast deductive techniques to realize fast abductive systems. In particular:

1. We provide new implementation methods for abduction using *model generation theorem provers* such as those in [Manthey and Bry, 1988; Fujita and Hasegawa, 1991]. Instead of finding some logical consequences of the given axioms, our methods generate some models of such formulas.
2. These methods are implemented *in parallel* on a parallel inference machine. Parallelization is an important source for realizing faster abductive systems.

3. Top-down information is incorporated in these bottom-up procedures. This is an extension of the *magic set* technique for deductive databases [Bancilhon *et al.*, 1986; Bry, 1990] to deal with abduction.

We use the parallel model generation theorem prover MGTP [Fujita and Hasegawa, 1991] that is implemented in the parallel logic programming language KL1 [Ueda and Chikayama, 1990]. Since the MGTP can be used for both *testing the (un)satisfiability* of an axiom set and *generating the minimal models* of a range-restricted axiom set, every function necessary for abduction can be realized on it. To this end, we show two different *program transformation* methods each of which converts an abductive problem into a model generation problem. The basic idea behind such a transformation has also been employed to compute *stable models* [Gelfond and Lifschitz, 1988] of general and extended (disjunctive) logic programs in [Inoue *et al.*, 1992a].

This paper is organized as follows. In Sections 2 and 3, abduction and the MGTP prover are summarized. Section 4 presents two realizations of parallel abductive systems using the MGTP. In Section 5, we evaluate these systems by applying them to a logic circuit design problem in [Maruyama *et al.*, 1988]. Some extension of the presented abductive systems and related work are discussed in Section 6.

## 2 Abduction

The definition of abduction we consider here is similar to that proposed in [Poole *et al.*, 1987]. An *abductive framework* is a pair  $(\Sigma, \Gamma)$ , where  $\Sigma$  is a set of formulas (the *background theory*) and  $\Gamma$  is a set of literals (the *hypotheses* or *abducibles*). Let  $G$  be a closed formula (the *goal*). A set  $E$  of ground instances of  $\Gamma$  is an *explanation of  $G$  from  $(\Sigma, \Gamma)$*  if

1.  $\Sigma \cup E \models G$ , and
2.  $\Sigma \cup E$  is consistent.

An explanation of  $G$  is *minimal* if no proper subset  $E'$  of  $E$  is an explanation of  $G$ . The computation of explanations of  $G$  from  $(\Sigma, \Gamma)$  can be seen as an extension of proof finding by introducing a set of hypotheses from  $\Gamma$  that, if they could be proved by preserving the consistency of the augmented theories, would complete the proof of  $G$ . Alternatively, abduction can be characterized by a consequence finding problem [Inoue, 1991], in which some literals are allowed to be hypothesized (or *skipped*) instead of being proved, so that new theorems consisting of only those skipped literals are derived at the end of deductions instead of just deriving the empty clause. In this sense, abduction can be implemented by an extension of a top-down, backward-chaining theorem-proving procedure. For example, Theorist [Poole *et al.*, 1987] and

SOL-resolution [Inoue, 1992] are extensions of the Model Elimination theorem proving procedure [Loveland, 1978].

However, there is nothing to prevent us from using a bottom-up procedure to implement abduction. In fact, we have developed an abductive reasoning system called APRICOT/0 [Ohta and Inoue, 1990], which consists of a forward-chaining inference engine and an ATMS [Reiter and de Kleer, 1987]. The ATMS is used to keep track of the results of inference in order to avoid both repeated proofs of subgoals and duplicate proofs on different hypotheses deriving the same subgoals.

Thus, the two reasoning architectures, top-down and bottom-up, are complementary, yet both have merits and demerits for computing abduction. As Inoue [1992] pointed out, SOL-resolution is *direct* in the sense that it is both sensitive to the given goal clause and restricted to searching only those formulas consisting of candidate hypotheses only. However, top-down reasoning may result in redundant proofs of subgoals. On the other hand, bottom-up reasoning eliminates redundancy, while it may prove subgoals unrelated to the proof of the given goal.

These facts suggest that it is promising to simulate top-down reasoning using a bottom-up reasoner, or to utilize cached results in top-down reasoning. The former simulation has been proposed for definite Horn databases as the Magic Set [Bancilhon *et al.*, 1986] or *upside-down meta-interpretation* [Bry, 1990] methods. As Stickel [1991] argues, this approach is better for abduction than the simulation of bottom-up reasoning by a top-down reasoner. This is because caching is more complicated and less effective for abduction since the search space for abduction is larger than that for deduction. Therefore, [Stickel, 1991] attempts the upside down meta-interpretation approach for abduction for Horn and non-Horn clauses. While Stickel does not consider the consistency of abductive explanations in his procedure, his approach has been extended to abduction for Horn clauses by incorporating consistency checking for a parallel version of APRICOT/0 in [Ohta and Inoue, 1992].

### 3 MGTP

In this section, we outline the model generation theorem prover MGTP [Fujita and Hasegawa, 1991; Inoue *et al.*, 1992a] on which our parallel abductive systems are based. The MGTP is a parallel and refined version of SATCHMO [Manthey and Bry, 1988], which is a bottom-up model generation theorem prover that uses hyperresolution and case-splitting on non-unit derived clauses.

Each *clause* in an axiom set  $\Sigma$  input to the MGTP is expressed in the form:

$$A_1, \dots, A_m \rightarrow C_{1,1}, \dots, C_{1,k_1} \mid \dots \mid C_{n,1}, \dots, C_{n,k_n}, \quad (1)$$

where  $A_i$ 's ( $1 \leq i \leq m$ ;  $m \geq 0$ ) and  $C_{j,k_j}$ 's ( $k_j \geq 1$ ;  $1 \leq j \leq n$ ) are atoms, and all variables are assumed to be universally quantified at the front of the clause. Clause (1)

represents the formula  $A_1 \wedge \dots \wedge A_m \supset (C_{1,1} \wedge \dots \wedge C_{1,k_1}) \vee \dots \vee (C_{n,1} \wedge \dots \wedge C_{n,k_n})$  in the standard notation of first-order logic. The left-hand side of  $\rightarrow$  is called the *antecedents* of the clause, while the right-hand side is called the *consequents* of the clause. When  $n = 0$ , the clause  $A_1, \dots, A_m \rightarrow$  is called a *negative clause* and means that whenever a ground instance of  $A_1 \wedge \dots \wedge A_m$  is true, there is a contradiction. When  $n = 1$  and  $k_1 = 1$ , the clause  $A_1, \dots, A_m \rightarrow C_{1,1}$  is called a *definite clause*. A *Horn clause* is either a definite clause or a negative clause.

In the following, an *interpretation* is defined as a set of ground atoms as usual, and is often called a *model candidate*. Given a current set  $\mathcal{M}$  of model candidates, the MGTP applies the following two operations to every model candidate  $M \in \mathcal{M}$  and generates the next set  $\mathcal{M}'$  of model candidates:

1. **(Model candidate extension)** If there is a non-negative clause of the form:

$$A_1, \dots, A_m \rightarrow C_{1,1}, \dots, C_{1,k_1} \mid \dots \mid C_{n,1}, \dots, C_{n,k_n}$$

and a substitution  $\sigma$  such that  $M \models (A_1 \wedge \dots \wedge A_m)\sigma$  and

$M \not\models (C_{i,1} \wedge \dots \wedge C_{i,k_i})\sigma$  for any  $i = 1, \dots, n$ ,

then  $M \cup \{C_{i,1}\sigma, \dots, C_{i,k_i}\sigma\}$  is in  $\mathcal{M}'$  for every  $i = 1, \dots, n$ .

2. **(Model candidate rejection)** If there is a negative clause of the form:

$$A_1, \dots, A_m \rightarrow$$

and a substitution  $\sigma$  such that  $M \models (A_1 \wedge \dots \wedge A_m)\sigma$ , then  $M$  is discarded.

Here, we call the process of obtaining such a substitution  $\sigma$  a *conjunctive matching* of the antecedents against elements in  $M$ . Note that this process does not need full unification if every clause is *range-restricted* [Manthey and Bry, 1988], that is, if every variable in the clause has at least one occurrence in its antecedents. Since every model candidate constructed by the MGTP in such a case contains only ground atoms, it is sufficient to consider one-way unification, i.e., matching, instead of full unification with costly occurs checking that does not allow circular bindings for variables. This is also a nice property for the implementation of the MGTP in KL1 [Ueda and Chikayama, 1990], because KL1 head unification is simply matching. The MGTP also improves efficiency by removing redundant conjunctive matching with a *ramified-stack* algorithm [Fujita and Hasegawa, 1991].

In the following, we assume that function symbols in the language are only constants and that the number of constants is finite. Given a set  $\Sigma$  of clauses and the initial set of model candidates  $\mathcal{M}_0 = \{\emptyset\}$ , the MGTP applies the above two operations to  $\mathcal{M}_0$  and generates  $\mathcal{M}_1$ . This process is repeated as long as  $\mathcal{M}_{n+1} \neq \mathcal{M}_n$  holds. If the MGTP cannot apply any operation to some model candidate set  $\mathcal{M}_\alpha$  (that is,

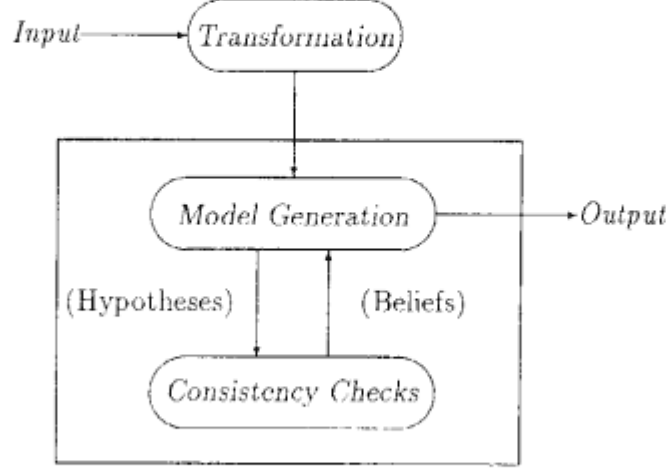


Figure 1: Abduction Architecture based on MGTP

$\mathcal{M}_\alpha$  is closed under the above two operations), it stops and returns  $\mathcal{M}_\alpha$ . If  $\mathcal{M}_\alpha = \emptyset$ , every model candidate has been rejected so that  $\Sigma$  is *unsatisfiable*. Otherwise, since each interpretation  $M \in \mathcal{M}_\alpha$  satisfies every ground clause from  $\Sigma$ ,  $M$  is a *model* of  $\Sigma$ . For example, if

$$\Sigma = \{ R(x) \rightarrow P(x) \mid Q(x), \quad R(x) \rightarrow S(x), \quad \neg R(A), \quad Q(x), S(x) \rightarrow \},$$

then  $\mathcal{M}_0 = \{\emptyset\}$ ,  $\mathcal{M}_1 = \{\{R(A)\}\}$ ,  $\mathcal{M}_2 = \{\{R(A), S(A)\}\}$ ,  $\mathcal{M}_3 = \{\{R(A), S(A), P(A)\}, \{R(A), S(A), Q(A)\}\}$ , and  $\mathcal{M}_4 = \{\{R(A), S(A), P(A)\}\} = \mathcal{M}_\alpha$ . Now, let us denote the set of minimal (in the sense of set inclusion of atoms) model candidates from  $\mathcal{M}$  as  $\min(\mathcal{M})$ . Then, the set of *minimal models* of  $\Sigma$  can be precisely given by  $\min(\mathcal{M}_\alpha)$  [Inoue *et al.*, 1992a].<sup>1</sup>

## 4 Abduction by Model Generation

Here, we introduce two realizations of abductive reasoning systems built on the MGTP.<sup>2</sup> Figure 1 illustrates the abstract architecture for these systems. In the following, we consider the first-order abductive framework  $(\Sigma, \Gamma)$ , where  $\Sigma$  is a set of range-restricted Horn clauses and  $\Gamma$  is a set of atoms (*abducibles*). Given an abductive framework  $(\Sigma, \Gamma)$ , these systems first transform the framework into a set of clauses

<sup>1</sup>A fixpoint semantics of the MGTP is formally discussed by Inoue and Sakama [1992].

<sup>2</sup>We have also developed several parallel abductive systems using the MGTP other than the two described in this paper. See [Inoue *et al.*, 1992b].

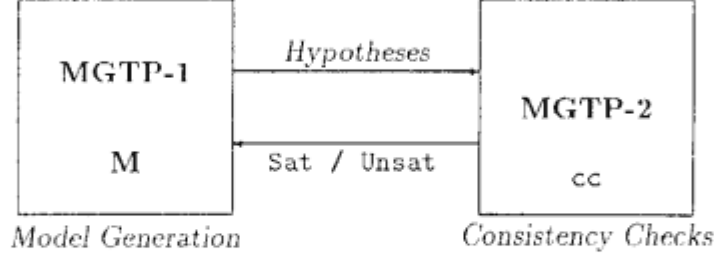


Figure 2: MGTP+MGTP

in a suitable form. Then, the MGTP is used for both (a) *Model Generation* of the transformed formulas and (b) *Consistency Checks* of some set of formulas augmented with hypotheses. These two functions correspond to the two conditions for each explanation  $E$  of  $G$  from  $(\Sigma, \Gamma)$  defined in Section 2: (1)  $\Sigma \cup E \vdash G$  and (2)  $\Sigma \cup E$  is consistent. Here, we convert the problem of finding  $E$  by (1) to some model generation problem (a). In this case, since  $\Sigma$  is a set of Horn clauses, any set  $\Sigma \cup E$  for any  $E$  from  $\Gamma$  is a set of Horn clauses and has the unique minimal model. In this way, the problem of finding explanations of  $G$  can be reduced to that of finding models of  $\Sigma$  that satisfy  $G$ . The MGTP, thus, outputs every model  $M$  of the transformed clauses such that  $M$  contains a minimal explanation of the given goal.

The two abductive systems below differ in how they transform the abductive framework  $(\Sigma, \Gamma)$ . The first one, the *MGTP+MGTP method*, is a co-operative system in which *Model Generation* and *Consistency Checks* communicate with each other. The second one, the *Skip method*, on the other hand, does not separate *Model Generation* from *Consistency Checks*, but expands some model candidates while pruning others. Both methods can be considered as natural extensions of a standard model generation method for deduction to model generation for abduction.

#### 4.1 MGTP+MGTP

The MGTP+MGTP method combines multiple MGTP's as illustrated in Figure 2.

In the MGTP+MGTP method, two kinds of MGTP's are combined: one MGTP is used for *Model Generation*, and the other MGTP's are used for *Consistency Checks*. Then, **MGTP-1** works as a forward-inference engine, while **MGTP-2** only tests the (un)satisfiability of the given axioms. In this system, each hypothesis  $H$  in  $\Gamma$  is represented by  $\text{fact}(H, \{H\})$ , and each *definite* clause in  $\Sigma$  of the form:

$$A_1 \wedge \dots \wedge A_m \supset C,$$



is transformed into a Horn clause of the form:

$$fact(A_1, E_1), \dots, fact(A_n, E_m) \rightarrow fact(C, cc(\bigcup_{i=1}^m E_i)), \quad (2)$$

where  $E_i$  is a set of hypotheses from  $\Gamma$  on which  $A_i$  depends, and the function  $cc$  is defined as:

$$cc(E) = \begin{cases} E & \text{if } \Sigma \cup E \text{ is consistent;} \\ \text{nil} & \text{otherwise.} \end{cases}$$

Especially, a positive unit clause  $\rightarrow C$  is transformed to  $\rightarrow fact(C, \emptyset)$ . In this case, since each transformed clause is a Horn clause, a unique, global model candidate (called a set of *beliefs* and denoted as  $\mathbf{M}$ ) is kept in each set of model candidates  $\mathcal{M}$  at every stage of computation. Each  $\mathbf{M}$  contains a set of atomic formulas in the form of  $fact(A, E)$  that represents a meta-statement that  $\Sigma \cup E \models A$ , and is stored in **MGTP-1**. Each time **MGTP-1** derives a new ground atom, the consistency of the combined hypotheses is checked by calling an **MGTP-2**.<sup>3</sup> Notice that **MGTP-1** never uses any negative clause in  $\Sigma$  for *Model Generation*.

In **MGTP-2**, each clause of  $\Sigma$  is used as is, and, in particular, the negative clauses are necessary. This MGTP is used just for *Consistency Checks*, and whenever a set of hypotheses  $E$  is received, it tries to find a model of  $\Sigma \cup E$ . If a model is found then it returns  $E$  itself, else nil is returned. Note that even when  $\Sigma \cup E$  is consistent we do not have to generate all of its minimal models.

The parallelism in this method comes from calling multiple **MGTP-2**'s at once. However, since every transformed clause is Horn, no case-splitting occurs so that **MGTP-1** may not be parallelized. Therefore, the effect of parallelization depends heavily upon how much consistency checking can be performed in parallel at once.

## 4.2 Skip

No matter how good the MGTP+MGTP method might be, the system consists of two different components. The possibilities for parallelization therefore remain limited. In contrast, the Skip method does not separate the inference engine from consistency checking, but realizes both functions in only one MGTP. In this method, the MGTP is used not only as an inference engine but also as a "generate-and-test" mechanism so that consistency checking is automatically performed. For this purpose, we can utilize the capability supplied by the MGTP to extend and reject model candidates. Therefore, multiple model candidates can be kept in distributed memories instead of keeping a single global belief set  $\mathbf{M}$  as in the MGTP+MGTP method. Thus, a great amount of parallelism can be obtained.

<sup>3</sup>Note that transformation (2) is similar to the method proposed by Stickel [1991], except that we additionally consider the function  $cc$  for consistency checking.

The most direct way to implement reasoning with hypotheses is as follows. For each hypothesis  $H$  in  $\Gamma$ , we supply a clause of the form:

$$\vdash H \mid \neg KH, \quad (3)$$

where  $\neg KH$  means that “ $H$  is not assumed to be true in the model”. Namely, each hypothesis is assumed either to hold or not to hold. Note here that we consider each literal  $\neg KH$  not as new formulas in a suitable modal logic, but as newly introduced *atoms* in a program, hence we are still within the classical first-order calculus. Then, we need the following axiom schema for dealing with  $\neg KH$ :<sup>4</sup>

$$\neg KH, H \rightarrow \quad \text{for every hypothesis } H. \quad (4)$$

The above schema is an integrity constraint saying that  $H$  cannot be true and disbelieved at the same time. In this way, each model candidate containing both  $H$  and  $\neg KH$  is rejected.

The above technique, however, may generate  $2^{|\Gamma|}$  model candidates, and is, therefore, often explosive for a number of practical applications. To reduce the number of generated model candidates as much as possible, we can use a method that delays case-splitting for each hypothesis.<sup>5</sup> That is, we do not supply any clause of the form (3) for any hypothesis of  $\Gamma$ , but, instead, introduce hypotheses when they are necessary. When abducibles  $H_1, \dots, H_n$  ( $n \geq 0$ ) from  $\Gamma$  appear in the antecedents of a clause in  $\Sigma$  as:

$$A_1 \wedge \dots \wedge A_l \wedge \underbrace{H_1 \wedge \dots \wedge H_n}_{\text{abducibles}} \supset C,$$

we transform the above Horn clause into a non-Horn clause:

$$A_1, \dots, A_l \rightarrow H_1, \dots, H_n, C \mid \neg KH_1 \mid \dots \mid \neg KH_n. \quad (5)$$

In this transformation, each hypothesis  $H_j$  in the antecedents is shifted to the right-hand side of the clause in the form of  $\neg KH_j$ . Moreover, each  $H_j$  is *skipped* instead of being resolved, and is added to consequent  $C$  of the rule since  $C$  becomes true whenever all  $A_i$ ’s and  $H_j$ ’s are true. This operation is a bottom-up counterpart of the **Skip** rule in the top-down approach defined for SOL-resolution in [Inoue, 1992]. Just like the transformation in (3), we need to supply the schema (4) for the Skip method. For example, suppose that  $\Sigma$  consists of the clauses:

$$\begin{aligned} & Person(x), Cold(x) \rightarrow Sneeze(x), \\ & Person(x), Hayfever(x) \rightarrow Sneeze(x), \\ & \rightarrow Person(Tom), \\ & Person(x), Cold(x), Hayfever(x) \rightarrow, \end{aligned}$$

<sup>4</sup>This was first introduced to regard *negation as failure* as a hypothesis in [Inoue et al., 1992a].

<sup>5</sup>The basic idea behind this delayed approach was first proposed for the processing of negation as failure with the MGTP in [Inoue et al., 1992a].

and the abducibles are  $\Gamma = \{ Cold(x), Hayfever(x) \}$ . Then,  $(\Sigma, \Gamma)$  is transformed to the following clauses  $\Delta$  by the Skip method:

$$\begin{aligned} & Person(x) \rightarrow Cold(x), Sneeze(x) \mid \neg K Cold(x), \\ & Person(x) \rightarrow Hayfever(x), Sneeze(x) \mid \neg K Hayfever(x), \\ & \rightarrow Person(Tom), \\ & Person(x) \rightarrow \neg K Cold(x) \mid \neg K Hayfever(x) \\ & \neg KH, H \rightarrow \quad \text{for every hypothesis } H. \end{aligned}$$

If we observe  $G = Sneeze(Tom)$ , then its explanations are obtained from  $\Delta$  by computing its minimal models containing  $G$ . Since two such minimal models are computed as:

$$\begin{aligned} M_1 &= \{ Person(Tom), Cold(Tom), Sneeze(Tom), \neg K Hayfever(Tom) \} \text{ and} \\ M_2 &= \{ Person(Tom), Hayfever(Tom), Sneeze(Tom), \neg K Cold(Tom) \}, \end{aligned}$$

we can get two explanations  $E_1 = \{ Cold(Tom) \}$  and  $E_2 = \{ Hayfever(Tom) \}$  by extracting the abducibles from  $M_1$  and  $M_2$ .

### 4.3 Upside-Down Meta Interpretation for Abduction

As discussed in Section 2, bottom-up abductive systems can enhance their efficiency by incorporating the goal information and by simulating top-down reasoning. Here, we present two program transformation methods based on *upside-down meta-interpretation* (UDM) approaches defined by [Bry, 1990; Stickel, 1991; Ohta and Inoue, 1992]. We first apply such a UDM transformation to the input clauses  $\Sigma$ , generating  $\Sigma'$ , then further transform  $(\Sigma', \Gamma)$  by using a transformation for the MGTP+MGTP method or the Skip method defined in previous subsections.

The first transformation, called the *simple UDM transformation*, transforms each definite clause of  $\Sigma$  in the form:

$$A_1, \dots, A_m \rightarrow C \tag{6}$$

into the following two clauses:

$$\begin{aligned} & goal(C), A_1, \dots, A_m \rightarrow C, \\ & goal(C) \rightarrow goal(A_i) \quad \text{for every } i = 1, \dots, m. \end{aligned}$$

In this method, the MGTP operations are applied for clause (6) only when  $goal(C)$  is present. Further,  $goal(C)$  invokes the subgoals of (6) by deriving every  $goal(A_i)$ . Thus, top-down reasoning is simulated in a breadth-first manner.

For each negative clause  $A_1, \dots, A_m \rightarrow$  in  $\Sigma$ , we can apply the above transformation used for definite clauses. Then, since the consequent of a negative clause is empty, we

supply  $\rightarrow \text{goal}(A_i)$  for every  $A_i$ . This means that any subgoal in every negative clause is evaluated. However, as Ohta and Inoue [1992] pointed out, many negative clauses are irrelevant to finding explanations of the given goal from the abductive framework in general. If we evaluate all the subgoals in all negative clauses, the UDM method cannot achieve speedups compared with non-controlled bottom-up abduction. To overcome this difficulty, we restrict the evaluation of negative clauses to those clauses relevant to the goal by using the *abstracted dependency analyzer* [Ohta and Inoue, 1992] that analyzes logical dependencies between the goal and negative clauses at the abstracted level.

The second transformation, called the *left-to-right UDM transformation*, simulates top-down reasoning in such a way that for clause (6) each subgoal  $\text{goal}(A_{i+1})$  ( $1 \leq i \leq m-1$ ) is invoked only after the previous subgoals  $\text{goal}(A_1), \dots, \text{goal}(A_i)$  have been solved. Thus, this method simulates ordered-linear resolution in a depth-first manner. In this method, each clause of form (6) is transformed to:

$$\begin{aligned} &\text{goal}(C) \rightarrow \text{goal}(A_1), \text{cont}_{k,1}(V), \\ &\text{cont}_{k,1}(V), A_1 \rightarrow \text{goal}(A_2), \text{cont}_{k,2}(V), \\ &\quad \vdots \\ &\text{cont}_{k,m}(V), A_m \rightarrow C, \end{aligned}$$

where  $k$  is the identifier of each clause  $C$  in  $\Sigma$ , and  $\text{cont}_{k,i}(V)$  keeps the substitutions  $V$  for variables appearing in  $A_1, \dots, A_{i-1}$ . As in the simple UDM transformation, the abstracted-dependency analysis is also employed for evaluating negative clauses in this transformation.

## 5 Evaluation

This section presents the evaluation of two abductive systems, the MGTP+MGTP method and the Skip method, by applying them to a design problem. The problem is to design a logic circuit that calculates the greatest common divisor (GCD) of two integers expressed in eight bits by using the Euclidean algorithm [Maruyama *et al.*, 1988]. The solutions are those circuits satisfying the given constraints on area and time. There are several kinds of knowledge on the design of circuits (about 120 clauses): datapath design knowledge (i.e., how to construct a GCD circuit by combining components) at the top level, component design knowledge (e.g., a subtractor can be constructed from the combination of a one's complement circuit and an adder) at the next level, and technology mapping rules (e.g., an adder can be constructed from a series of some CMOS standard cells) at the low level. The problem can be represented as abduction, in which we assume that some combination of components may satisfy all constraints. Thus, if hypotheses derive a contradiction with the background knowledge, we see that the sub design violates some constraints.

Table 1: Evaluation of Abductive Systems

Table 1-1: Normal Transformation

Problem		System type	
		MGTP+MGTP	Skip
<i>Circuit</i>	(limit 300-40)	4.0/ 7.5(64)	7.2/ 13.7(64)
	(limit 400-50)	31.2/ 3.3(64)	23.3/ 19.4(64)
	(limit 500-60)	54.3/ 2.6(64)	26.1/ 20.6(64)

Table 1-2: Simple UDM Transformation

Problem		System type	
		MGTP+MGTP	Skip
<i>Subtractor</i>	(limit 500-60)	1.1/ 1.3(64)	0.6/ 1.2(64)

Table 1-3: Left-to-Right UDM Transformation

Problem		System type	
		MGTP+MGTP	Skip
<i>Subtractor</i>	(limit 500-60)	0.8/ 1.0(16)	12.7/ 3.1(64)

[Notation]  $T / R(PEs)$

$T$  : The smallest execution time (sec)

$R$  : The speedup ratio of  $T$  to the time obtained with a single PE (times)

$PEs$  : The number of PEs used when  $T$  is obtained (number)

Table 1 shows the experimental result of two abductive systems at run time on the PIM/m parallel inference machine developed at ICOT. The area-time limits are set in three ways (300-40ns, 400-50ns and 500-60ns). In order to evaluate two UDM methods, we also solved a subproblem (the design of subtractors) of the entire problem (the design of GCD circuits).<sup>6</sup>

As shown in the table, the run time for designing subtractors in each system is, as expected, much shorter than that for designing GCD circuits, since the reasoning is directed to the given subgoal. However, for the Skip method, the simple UMD transformation works better than the left-to-right UDM transformation, indicating that the left-to-right simulation of top-down reasoning increases the sequential processing more than the breadth-first manner.

Figure 3 shows the run time graph for the design of GCD circuits with a 500-60ns limit, which is obtained by varying the number of available processor elements (PEs) between 1 and 64 on PIM/m. Figure 4 displays the speedup ratio for the same problem when running two abductive systems. All reasoning tasks split with disjunctions are automatically allocated to the available number of processors.

As shown in these figures, the Skip method provides better parallelism as well as faster abductive reasoning than the MGTP+MGTP method.

## 6 Final Remarks

We have presented several new program transformation techniques for fast, parallel and bottom-up abduction. First, we have converted the Horn abductive problem with consistency checking into model generation problems in two ways. Second, we have applied the two kinds of upside-down meta-interpretation transformations to abduction to incorporate top-down information. Although we need to further investigate how to avoid possible combinatorial explosion in constructing model candidates for the Skip method, we conjecture that the Skip method will be the most promising from the viewpoint of parallelism.

For related work on computing abduction using a model generation theorem prover, Denecker and Schreye [1992] recently proposed a proof procedure for *object-level abduction* defined in [Console *et al.*, 1991]. But, in contrast to us, they compute the models of the only-if part of a completed program that is not range-restricted in general even if the original definite clauses are range-restricted. To this end, they have to extend the model generation method by incorporating complex term rewriting techniques, while we can use the original MGTP without change in the Skip method. Furthermore, their abduction does not consider the consistency of explanations.

<sup>6</sup>In order to avoid possible combinatorial explosion caused by many redundant model candidates containing non-minimal explanations, we introduced some negative clauses that can be generated automatically by analysis in the Skip method. See [Inoue *et al.*, 1992b] for details.

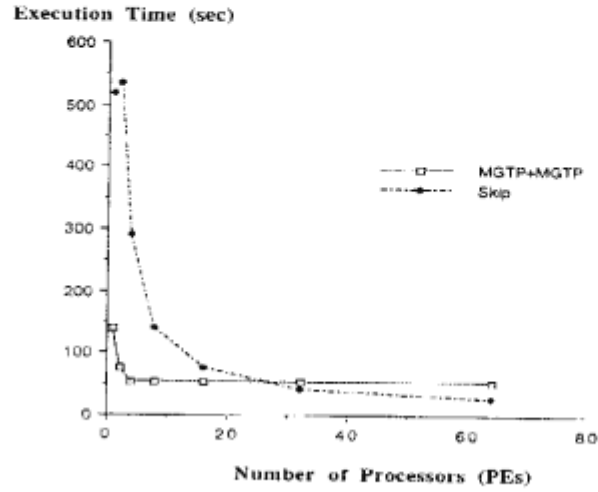


Figure 3: Execution Time (*Circuit: limit 500-60*)

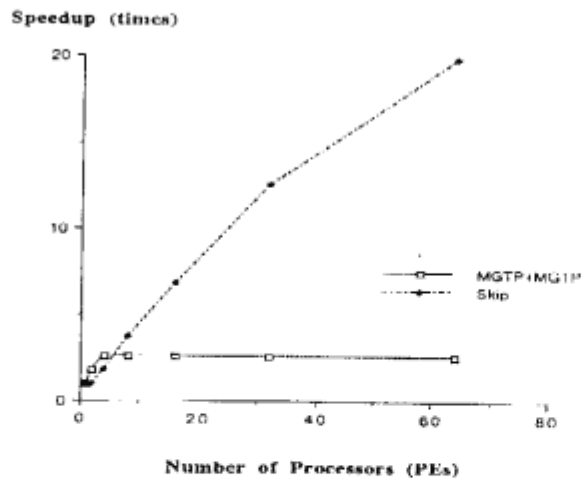


Figure 4: Speedup Ratio (*Circuit: limit 500-60*)

While we restricted the abductive framework  $(\Sigma, I')$  to a pair of Horn clauses and atomic abducibles, we can consider an extension of abduction which captures non-monotonic and default reasoning. Then, another important advantage of the Skip method is that it may easily be combined with *negation as failure* so that knowledge bases can contain both abducibles and negation-as-failure formulas as in the framework of [Kakas and Mancarella, 1990]. Notice that the transformation of abducibles by (5) is not dual to the transformation of negation as failure in [Inoue *et al.*, 1992a]. For a negation-as-failure formula *not P*, the current model candidate is split into two, one containing  $\neg KP$  and the other containing  $KP$ , where an atom  $KP$  means that *P should be derived*. Here, for an abducible *H*, we generate two model candidates, one containing *H* itself and the other containing  $\neg KH$ . This asymmetry comes from the fact that each abducible can be added to model candidates without imposing the condition that it should be derived so that we do not have to test the stability of each abducible in the fixpoint (that is, we need not test whether *P* is actually in *M* for each  $KP \in M$ ). Therefore, when a negation-as-failure formula *not H* mentions an abducible *H*, it should be split into  $\neg KH$  and *H*. This extension of the abductive framework by incorporating negation as failure will be reported in detail elsewhere.

Finally, efforts should be also devoted to investigating extensions of bottom-up first order abduction to deal with non range-restricted, non-Horn clauses and literal abducibles, which can be dealt with by a top-down approach like SOL-resolution [Inoue, 1992]. An example of bottom-up abduction for non-Horn clauses without consistency checking can be found in [Stickel, 1991], which uses contrapositives in the form of definite clauses. This technique may be incorporated for the MGTP+MGTP method with consistency checking. One difficulty lies in the fact that, since the definite transformation using the meta predicate *fact* does not involve any case-splitting as is the case in the MCTP+MGTP method, it does not seem to be promising from the viewpoint of OR-parallelism on the MGTP.

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