ICOT Technical Report: TR-679

TR-679

A Logical Analysis of Relevance in Analogy

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August, 1991

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Abstract: This paper treats a general type of analogy which is described as follows: when two objects, B (called the *base*) and T (called the *target*) share a property S (called the *similarity*), it is conjectured that one T satisfies another property P (called the *projected property*) which the other B satisfies as well. This type of analogy is analyzed formally, and a logical relevance, called the *illustrative criterion*, w.r.t. T,B,S,P under a given theory $\mathcal{A}$ is extracted.

In the study of analogy, the following have been central problems: 1) what object should be selected as a base w.r.t a target, 2) which property is important in analogy among properties shared by two objects, and 3) what property is to be projected w.r.t. a certain similarity. The criterion exposed here should shed light on a way to solutions for them.

1 Introduction

When we explain a process of reasoning by analogy, we may say that "An object T is similar to another object B in that T shares a property S with B and B satisfies another property P. Therefore, T satisfies P, too", or it may be expressed more formally by the following schema.

$$\frac{\begin{array}{l} S(B) \wedge P(B) \\ S(T) \end{array}}{P(T)} \tag{1}$$

Here, T will be called the *target*, B the *base*, S the *similarity* between T and B, and P the *projected property*.

Nevertheless, the above description of the process of analogy is insufficient. Researchers studying analogy have come to recognize the necessity of revealing some implicit condition which influences the process but does not appear in the above schema. T.R.Davies *et al.* [3]

give intuitive examples which show the existence of such an implicit condition. Here, we take another example and review the problem.

Example[1]: Brutus feels pain when he is cut or burnt. Also, Tacitus feels pain when he is cut. Therefore, when Tacitus is burnt, he would...

Both Brutus and Tacitus feel pain when they are cut (as a similarity), but we could not infer that Tacitus is strong just because Brutus is strong (as a projected property). However, we may infer that Tacitus feels pain when he is burnt just because Brutus feels pain when he is burnt (as another projected property). The point is the fact that we prefer the latter reason to the former with respect to the similarity though neither has any difference in applying the above schema. It clearly suggests that the plausibility of the conclusion depends on some implicit condition that is not provided in the premise of the schema and that relates a similarity with a projected property. Such an implicit condition should also be related to the target and the base, because a similarity is a common property of a target and a base, and because the object that is the base defines possible candidates for the projected property. To reveal such an implicit condition which justifies some analogical inference is very important, because it prevents an unrestricted superficial application of the analogical schema from yielding useless conclusions.

Much work on analogy has assumed a base to be given, and also a similarity to be given without clarifying the relation between the similarity and the projected property, or have defined a similarity independently of the property projected [9, 4, 5, 6]. For instance, Winston's program works based on a similarity measure which depends on counting equivalent corresponding attributes in a frame, this means that the similarity is decided a priori independent of the projected property. We should consider similarity in the context of a projected property when the projected property is given. There is no doubt that what is the similarity depends on what property is projected, and, moreover, the similarity influences selection of the base, while a base for a target circumscribes candidates of a similarity. Thus, an overall exploration of relevance w.r.t. a target, a base, a similarity and a projected property is needed. The work proposed by T.R.Davies *et al* [3] is also done from such a motive. Their solution, however, does not completely satisfy us in that, according to their approach, once we give the implicit condition (as their criterion) to a theory, analogy collapses into deduction. This will be against our intuition ("analogy" is not deductive). Here, we seek another criterion w.r.t. a target, a base, a similarity, a projected property and a given theory which leaves analogy non-deductive. In this research, we take an approach to this goal in which we analyze analogy under the following two natural premises, by using classical first-order logic.

Premise 1: "Analogy is done by projecting properties from a base onto a target."

Premise 2: "The target is not a special object."

[1]This example originates from a famous science fiction story[7].

2 A logical analysis of relevance and illustrative criterion

The objective of this section is to analyze analogy as formally as possible and to obtain logical relevance w.r.t. a target, a base, a similarity, a projected property and a theory.

In this paper, we use standard formal logic and its notations. An n-ary *predicate* U is generally expressed by $\lambda x Q$, where x is a tuple of n object variables, Q a formula in which any object variable except x does not occur *free*. If U is not (semantically) equivalent to any m-ary predicate (m < n), U will be called *argument-sensitive*[2]. If t is a tuple of n terms, $U(t)$ stands for the result of replacing each occurrence of (elements of) x in Q by (each corresponding element of) t simultaneously. *Knowledge* means a *closed* universal formula in this paper and is expressed by a conjunction of *clauses* or a set of clauses according to its context (the former expression is called a *formula expression* and the latter a *clause-set expression*). For instance, if Γ is knowledge, Γ will be expressed by a conjunction of clauses when it appears on the right side of $\vdash$ and by a set of clauses when on the left.

Let knowledge $\mathcal{A}$ be a theory (a set of proper axioms), T and B terms (or a tuple of terms), and P a predicate. Let S be a argument-sensitive predicate in which B does not occur. Then, the analogy mentioned previously is expressed as follows.

$$\mathcal{A} \vdash S(B) \wedge P(B), \tag{2}$$

$$\mathcal{A} \vdash S(T), \tag{3}$$

Of the above meta-sentences, the former means that the base can be shown to have a certain property S (a similarity) and another property P (a projected property) from $\mathcal{A}$, and the latter means that the target can be shown to share the property S. Moreover, in general, analogy is not deductive.

$$\mathcal{A} \not\vdash P(T). \tag{4}$$

As analogy infers $P(T)$ under these, it implies that some knowledge F_A is assumed in the premise of (4). That is,

$$\mathcal{A}, F_A \vdash P(T). \tag{5}$$

Now, as analogy is made by projecting properties from the base onto the target (Premise 1), the following would hold. For some knowledge f_B:

$$\mathcal{A} \vdash f_B, \quad \text{and,} \quad F_A = f_B(T), \tag{6}$$

where $f_B(T)$ is the result of replacing every occurrence of (elements of) B with (each corre-

[2]**Ex.** Neither $(\lambda x, y\ P(x))$*nor*$(\lambda x, y\ P(x) \vee Q(x,y) \wedge \neg Q(x,y))$ are argument-sensitive, because each of them has a semantically equivalent unary predicate, $(\lambda x P(x))$.

sponding element of) T in f_B. Thus, the essential information newly obtained by analogy is f_B in (6) rather than the explicit projected property P. Moreover, analogical reasoning must be consistent:

$$\mathcal{A} \cup f_B(T) \text{ is consistent.} \tag{7}$$

Now, T is not special (Premise 2), that is, some object x must be conjectured to satisfy P if x satisfies S. As the same arguments to T can be applied to x when $S(x)$ is added to $\mathcal{A}$,

$$\mathcal{A} \vdash f_B, \quad \text{and} \quad \mathcal{A}, S(x), f_B(x) \vdash P(x), \tag{8}$$

which is transformed equivalently to

$$\mathcal{A} \vdash f_B, \quad \text{and} \quad \mathcal{A} \vdash (\forall x f_B(x) \wedge S(x) \supset P(x)) \tag{9}$$

because $\mathcal{A}$ is closed.

Now, the last clue to clarifying analogy is also extracted from the premise that the target is not special. This premise implies that some class of properties satisfied by base B is projected to an arbitrary (similar) object which shares similarity S with B. From this observation, it will be concluded that a reason (justification) for an analogical inference (that is, a certain non-deductive projection) is based just on the fact that an object as the target shares similarity with the base. Then, what class of properties is, in some sense, *justified* in being projected onto an object t such that $\mathcal{A} \vdash S(t)$ when $\mathcal{A} \vdash S(B)$ and, as the result, yields non-deductive conclusions? In the remainder of this section, we will consider the problem. As preparation for this, we define a few meta relations.

Definition 1: to explain. Let α be a set of clauses and β a formula. α *explains* β, written as "$\alpha \vdash_{exp} \beta$", if β is a logical consequence of α ($\alpha \vdash \beta$) and α is a minimal set (i.e. if we remove any subset from α, β is no longer a logical consequence of the remainder α). □

Definition 2: explanation. Let $\mathcal{A}$ and F be a set of clauses and β a formula. F is an *explanation* of β with $\mathcal{A}$, written by "$F \vdash^{\mathcal{A}}_{exp} \beta$", if F is a subset of $\mathcal{A}$ ($F \subseteq \mathcal{A}$) and F explains β ($F \vdash_{exp} \beta$). □

Then, $\mathcal{A} \vdash S(B)$ is equivalent to the following:

For some F^S_B:

$$F^S_B \vdash^{\mathcal{A}}_{exp} S(B). \tag{10}$$

This meta-sentence shows that a subset F^S_B of $\mathcal{A}$ is a minimal set of axioms from which $S(B)$ follows. Further, we define a minimal set of theorems of such F^S_B from which $S(B)$ follows.

Definition 3: abstract explanation. Let $\mathcal{A}$ and α be a set of clauses and β a formula. α is an *abstract explanation* of β with $\mathcal{A}$, written as "$\alpha \vdash^{\mathcal{A}}_{abst} \beta$", if, for some F, $F \vdash^{\mathcal{A}}_{exp} \beta$, $F \vdash \alpha$

and $\alpha \vdash_{exp} \beta$. □

Then, the following is also apparently equivalent to $\mathcal{A} \vdash S(B)$.

For some f_B^S:

$$f_B^S \vdash_{abst}^{\mathcal{A}} S(B). \tag{11}$$

That is, we can handle (11) instead of $\mathcal{A} \vdash S(B)$ without loss of information. Now, we consider justified projection among objects satisfying similarity S. Under our weak premises, we would have few candidates for such projection. Assume that all we know about a target is just the fact that the target satisfies S. Justification of projection in analogy is given just from that fact. The fact that the target satisfies S follows sufficiently if we assume that the target satisfies a f_B^S[3] in (11)[4]. In this sense, the projection of a f_B^S is justified unless it yields contradiction to $\mathcal{A}$ ($\mathcal{A} \cup f_B^S(T)$ is consistent). It would be difficult to justify projections of other class of properties in any sense without additional premises because of the following: 1) In the case that the result of a projection of a certain property does not follow from $\mathcal{A}$ and a $f_B^S(T)$, the property is unnecessary in yielding the fact that the target satisfies S and there is no sign that the target would satisfy the property (Remember that all we know about a target is just the fact that the target satisfies S). 2) Otherwise, projection of the property is unnecessary, as the projection of the f_B^S will be sufficient.

From this observation, it turns out that only f_B^S would be justified in being projected. Now, it is only the knowledge involving the occurrence of B that is actually transformed by replacing B with T. Let F be a set of clauses and O a term (or, a tuple of terms). By $F|_O$, we mean a set of all clauses in F in which O occurs (or, some elements of O occur). Then, the consequence of the above arguments can be described as follows:

$$f_B = f_B^S|_B. \tag{12}$$

From (9) and (12),

$$\mathcal{A} \vdash (\forall x f_B^S|_B(x) \land S(x) \supset P(x)) \tag{13}$$

is obtained. (Note that $\mathcal{A} \vdash f_B^S|_B$ by definition.) It is simplified to

$$\mathcal{A} \vdash (\forall x f_B^S|_B(x) \supset P(x)).^5 \tag{14}$$

[3]Accurately speaking, as f_B^S is not a property, $\lambda x f_B^S(x)$ is correct.

[4]The proposition shown later guarantees that.

[5]Let R be the compliment of $f_B^S|_B$ in f_B^S, that is, let R be a set of the clauses in which B does not occur in f_B^S. As f_B^S is an abstract explanation of $S(B)$ and $f_B^S = f_B^S|_B \cup R$, it yields $f_B^S|_B, R \vdash S(B)$ and S itself does not involve B, therefore, $R \vdash (\forall x \cdot f_B^S|_B(x) \supset S(x))$. Because $\mathcal{A} \vdash R$, $\mathcal{A} \vdash (\forall x\ f_B^S|_B(x) \supset S(x))$.

Consequently, we summarize the above results (2), (3), (7), (12), (14) as the following criterion.

Definition 4: illustrative criterion. T and B are (tuples of) object constant terms. S is an argument-sensitive predicate in which none of B occur, and P is a predicate. $\mathcal{A}$ is a set of closed clauses (knowledge). Then, the 5 tuple, $< T, B, S, P, \mathcal{A} >$, satisfies *the illustrative criterion* if all of the following conditions are satisfied:

i) *The outward similarity:* $\mathcal{A} \vdash S(T) \wedge S(B)$,

ii) *The causal relevance:* For some f^S_B s.t. $f^S_B \vdash^{\mathcal{A}}_{abst} S(B)$,

ii-a) *causal implicability:* $\mathcal{A} \vdash (\forall x f^S_B|_B(x) \supset P(x))$

ii-b) *consistency:* $\mathcal{A} \cup f^S_B|_B(T)$ is consistent,

where $f^S_B|_B(T)$ is the result of replacing every occurrence of B with T in $f^S_B|_B$. □

Intuitively, i) The *outward similarity* shows that both T and B can be shown to share a property S from $\mathcal{A}$. ii) The *causal relevance* shows that a cause $f^S_B|_B$ of a phenomenon, $S(B)$ ("B satisfies S"), involves a cause of a phenomenon, $P(B)$ ("B satisfies P"), and it is consistent even if T has the former cause, $f^S_B|_B$ (T).

$\lambda x f^S_B|_B(x)$ is called the *inward similarity* because $f^S_B|_B(T)$ does not generally follow from $\mathcal{A}$ (on the other hand, $\mathcal{A} \vdash f^S_B|_B(B)$), while S is called the *outward similarity* because it can be shown to be shared by B and T.

The reason that the above criterion is called *illustrative* and is based on the fact that there exists at least one object satisfying both P and S that can be explained by the reason of satisfying a certain causal property $f^S_B|_B$. That is, satisfying S is circumstantial evidence of satisfying $f^S_B|_B$ and satisfying P can be proved from satisfying $f^S_B|_B$, which is justified (illustrated) with at least one thing, a base.

When 5 tuple $< T, B, S, P, \mathcal{A} >$ satisfies the illustrative criterion, $P(T)$ is called *justified illustratively*, and, in this case, when $P(T)$ is obtained by an analogy, the analogy is called *illustrative* and each corresponding couple of B and T are called *illustratively similar.*

The following holds.

Proposition. When 5 tuple $< T, B, S, P, \mathcal{A} >$ satisfies the illustrative criterion, denoting $\lambda x f^S_B|_B(x)$ as an inward similarity, the following holds.

$$f^S_B|_B(T), A \vdash P(T)$$ □

The above theorem says that, once $f^S_B|_B$ is projected onto T (that is, $f^S_B|_B(T)$ is assumed), $P(T)$ is deducible from $\mathcal{A}$ even if $\mathcal{A} \not\vdash P(T)$.

This criterion, to be satisfied by analogy, was obtained from formal arguments. What meaning does such a formal analogy have? The next section gives the answer.

3 Intuitive meaning of the criterion

Consider the traditional philosophical problem of other minds. While I know that I have a mind because of my experiences, I do not have any experience of your experiences. Then, how do I know that you also have a mind? Apparently, though the only evidence we have on which to base our inference is the external observation of others, we still seem to be able to reach the proper conclusion. When we need to infer something about others, by imagining ourselves in their shoes and by simulating them, we sometimes find an explanation of how and why they are what they are, and can conjecture unknown properties which they would satisfy, for instance, their present state, character, changes of mind, and the purposes of their past and future actions. Such reasoning may be divided into the following four steps: 1) to think about an observation of an unknown domain ($S(T)$) as if it were a case of a well-known domain ($S(B)$), 2) to extract implicit properties ($f_B^S|_B$) from the well-known domain which are necessary to explain and understand the observation ($f_B^S \vdash_{exp} S(B)$), 3) to map the extracted knowledge into the unknown domain ($f_B^S|_B(T)$), and then to deduce various plausible conclusions about the unknown domain from the mapped knowledge and general knowledge ($f_B^S|_B(T), \mathcal{A} \vdash P(T)$). Therefore, such reasoning can be considered as a type of illustrative analogy, where the base case is ourselves, the target is the other person, the (outward) similarity is the fact that we can cause the same phenomenon, and the projected properties are, essentially, facts about the base case which are needed as some premises in the explanation of the phenomenon, or logical consequences of given knowledge and the projected facts about the base.

One of general objectives of the research of computational analogy will be to make a system which, given a theory including various cases, can answer a query, "Does T satisfy P ($P(T)$?)" by making use of analogy. For this goal, we need to solve some crucial problems: how we should retrieve a proper and *similar* base case and how we should focus on a *relevant* property among so many similarities. In general, the following have been central problems: 1) what object should be selected as a base w.r.t a target, 2) which property is important in analogy among properties shared by two objects, and 3) what property is to be projected w.r.t. a certain similarity. The criterion exposed here would shed light on a way to find solutions for them. An object B for a given target T (an object T for a given B) is *similar* when B,T satisfies an illustrative criterion. Also, a property S for a given P (a property P for a given S) is *relevant* (or, *important*) when S,P satisfies the illustrative criterion.

4 Example

Various types of reasoning can be treated as illustrative analogy and abundant application of illustrative analogy can be considered: recognition of other agent's purpose, discovery by analogy and explanation of why and how similarity changes according to the problems to be solved. Here, we show one example (We will show more in a longer version of this paper).

Example: Discovery by analogy. Illustrative analogy is used to predict unknown properties w.r.t. our solar system, based on the fact that a planet revolves around the sun. Under $S = Revolves$, $T = Planet, Sun$, the issue is to search P and B which satisfy the illustrative criterion. Let the proper axiom $\mathcal{A}$ involve the following knowledge of our solar system: "A planet revolves around the sun", "The sun is apart from the planet", "The sun is not connected to anything" and "The planet is not connected to anything", i.e.

$$Revolves(Planet, Sun), \quad \text{(E1.1)}$$
$$ApartFrom(Sun, Planet), \quad \text{(E1.2)}$$
$$\forall x \neg Connects(Sun, x), \quad \text{(E1.3)}$$
$$\forall x \neg Connects(Planet, x). \quad \text{(E1.4)}$$

First of all, search for a target which can be shown to satisfy the similarity is done, that is, it is a search for a tuple $< x, y >$ which satisfies $Revolves(x, y)$ (which can be done by ordinal *deductive query-answer systems*). Here, we assume that $\mathcal{A}$ also involves the following knowledge on experiences of Mr. N of revolving a stone with string: "a stone revolves around and apart from Mr. N", "The stone is not supported", "Mr N is connected to a piece of inextensible string which is connected to the stone", "Mr. N draws the string" and "When Mr. N attracts something unsupported, if it remains apart from Mr. N, then it revolves around Mr. N", which are described as

$$Revolves(Stone, N), \quad \text{(E1.5)}$$
$$ApartFrom(N, Stone), \quad \text{(E1.6)}$$
$$\neg Supported(Stone), \quad \text{(E1.7)}$$
$$Connects(N, String), \quad \text{(E1.8)}$$
$$Connects(Stone, String), \quad \text{(E1.9)}$$
$$Inextensive(String), \quad \text{(E1.10)}$$
$$Draws(N, String), \quad \text{(E1.11)}$$
$$(\forall x \neg Supported(x) \wedge ApartFrom(N, x) \wedge Attracts(N, x) \supset Revolves(x, N)). \quad \text{(E1.12)}$$

Additionally, $\mathcal{A}$ involves some general (physical) rule: "When x is connected to inextensible z and y is connected to z, if x draws z then x attracts y" and "If x is not connected to anything, x is not supported", which are described as

$$(\forall x, y, z Inextensible(z) \wedge Connects(x, z) \wedge Connects(y, z) \wedge Draws(x, z) \supset Attracts(x, y)), \quad \text{(E1.13)}$$
$$(\forall x \exists y \neg Connects(x, y) \supset \neg Supported(x)). \quad \text{(E1.14)}$$

Then, for the query "What is a tuple $< x, y >$ s.t. $Revolves(x, y)$?", let the answer be $< Stone, N >$, that is, $B = Stone, N$. The result reveals correspondence between the target and the base, that is, in this analogy, *Planet* corresponds to *Stone* and *Sun* corresponds to N.

Second, by explaining why the stone revolves around Mr. N, candidates of inward similarity are extracted. The candidates occur in such an explanation, and the following abstract property of the base satisfies the illustrative criterion as an inward similarity($f_B^S|_B$) [6]:

$$\begin{gathered}(\forall x \neg Supported(x) \wedge ApartFrom(N, x) \wedge Attracts(N, x) \supset Revolves(x, N)),\\ \neg Supported(Stone),\\ ApartFrom(N, Stone),\\ Attracts(N, Stone).\end{gathered}$$

Finally, by projecting this to the target, the following non-deductive conclusion ($P \subseteq \lambda x(f_B^S|_B(x))$) is obtained, that is, from $f_B^S|_B(T)$,

$$\begin{gathered}(\forall x \neg Supported(x) \wedge ApartFrom(Sun, x) \wedge Attracts(Sun, x) \supset Revolves(x, Sun)),\\ Attracts(Sun, Planet).\end{gathered}$$

This knowledge says that "When the sun attracts something unsupported, if it remains apart from the sun, then it revolves around the sun" and that "The sun attracts the planet". This suggests that the sun might be able to attract the planet without having any connection to it, which might lead to the discovery of universal gravitation.

Also, note that, even if $\mathcal{A}$ involves $Englishman(N)$, this property would not be projected into the sun (i.e. $Englishman(Sun)$) because the property will be unnecessary in explaining why the stone revolves around Mr. N.

5 Conclusion and Remarks

- **What is suggested by the results obtained here?**

  Note that necessity is preserved at each step to getting the illustrative criterion in the second section. That is, once we assume the two premises for analogy used in this paper, it seems to be **an inevitable conclusion** that an analogy which infers $P(T)$ from $S(T), S(B), P(B)$ satisfies the illustrative criterion (We must, however, pay attention to the possibility that translation of the premises into a formal system might yield some gaps).

  Therefore, to examine whether a system of analogy satisfies the criterion will be interesting. Because, if it does not satisfy, it suggests that the system ignores the premises in some points or assumes other strong premises.

[6] A more specific candidate, $\{(E1.6) \sim (E1.12)\}$, does not satisfy the consistency condition.

- **What new system will come from the results obtained here?**

  Many novel systems are suggested by the results, for example, a system, without given "important" similarities a priori, adaptively decides similarities depending on a given problem to be solved.

- **Open problems**

  a) Though this research might shed light on the way to realize a general analogical inference system, here we show a mere **specification** which must (possibly) be satisfied by a logical system for analogy. A search algorithm for tuples which satisfies the illustrative criterion remains to be explored in the future.

  b) In the case that there are multiple explanations, especially when different bases yield different explanations, their illustratively justified conclusions could contradict with each other. The situation would not be strange at all, because we often find that analogies based on different bases lead to different conclusions. Though this would be true, we should not ignore the fact that we prefer one explanation more than others according to the situation. The problem of which explanation is most preferable to us is open.

Acknowledgments

This paper is a revised version of early works[1, 2]. I am grateful to Natsuki Oka, Katsumi Inoue, Dr. Makoto Haraguchi and Dr. Koichi Furukawa for their useful comments.

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