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Relative Plausibility based on Model Ordering: Preliminary Report

by

K. Satoh

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ICOT

Mita Kokusai Bldg. 21F
4-28 Mita 1-Chome
Minato-ku Tokyo 108 Japan

(03) 456-3191～5
Telex ICOT J32964

Institute for New Generation Computer Technology

Relative Plausibility based on Model Ordering: Preliminary Report

Ken Satoh

ICOT Research Center,

1-4-28, Mita, Minato-ku, Tokyo 108, Japan

csnet: ksatoh%icot.jp@relay.cs.net

uucp: {enea,inria,kddlab,mit-eddie,ukc}!icot!ksatoh

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Abstract

This paper presents a formalization of relative plausibility. In stead of giving numerical value to propositions such as probability theory, we give an ordering over propositions and derive results which are true in the most plausible models. A formalism is based on a model ordering which is an extension of circumscription. We show a semantic and syntactic definition and examples of derivation.

1 Introduction

In real life, we seldom get complete knowledge to make the right decision, and we often cannot wait until enough knowledge is gathered. In that case, we have to draw a conclusion based on some heuristics to complement unknown information. Much research has been conducted to formalize this phenomenon. We can divide the research into two areas: the numerical approach and the non-numerical approach.

The major research from the numerical approach to uncertainty is the probability theory, Shafer's Evidence Theory, and Zadeh's Possiblity Theory. This approach represents uncertainty by attaching a numerical value to a proposition and defining operation over those values. If this approach works quite well, it will be the most powerful tool for uncertainty. However, there are some problems in practice: for example, it is not always easy to attach a numerical value to a proposition, and people are reluctant to give a numerical value for likelihood. See discussion on that issue in the introduction of [Halpern87].

On the other hand, default reasoning [McCarthy80, Reiter80] has been intensively investigated as a non-numerical approach to uncertainty. The idea of default reasoning is intuitively based on "negation as ignorance"; that is, if there is no positive known information of a proposition, P, then $\neg P$ is assumed. As McCarthy points out in [McCarthy86], this idea can capture some qualitative notion of the numerical approach to uncertainty. However, this idea is not strong enough to capture all qualitative notions which can be expressed in the numerical approach. For example, we are not sure how to capture relative plausibility such as "P is more plausible than Q".

This paper attempts to extend default reasoning to capture the notion of relative plausibility. The notion of relative plausibility is an extension of default reasoning because we can rephrase the basic idea of default reasoning, that is, "negation as ignorance" as "$\neg P$ is more plausible than P".

In our formalism, we give ordering over formulas to represent relative plausibility. For example, suppose that a symptom of a patient is S_1 and we have the following information.

- A disease, D_2, is more plausible than a disease, D_1.
- If S_1 is found, then it is likely that the patient suffers from D_1 or D_2.

We represent the above information of relative plausibility by

$$\{D_1 \prec D_2, S_1 \supset \neg(D_1 \vee D_2) \prec S_1 \supset (D_1 \vee D_2)\}$$

. We introduce a new model ordering, which is an extension of circumscription and we regard the above sentence as the meta-statement to decide model ordering. Then we define the most plausible models which are the maximal models based on the above model ordering. We give a syntactic definition from which we derive results which are true in the most plausible model. In the above example, we can conclude that only S_1 and D_2 are true in the most plausible models.

2 Semantic Definition of Relative Plausibility

Before defining model ordering, we first define the syntax of meta-statements to express relative plausibility. A set of meta-statements of relative plausibility consists of expressions of the form:

$$E(P,x) \prec E'(P,x),$$

where $E(P,x)$ and $E'(P,x)$ are wffs in which all free variables are in a tuple of variables x and all predicates are in a tuple of predicates P. The intended meaning of the above expression is "$E'(P,x)$ is more plausible than $E(P,x)$". We write $RP = \{E_1(P,x_1) \prec E'_1(P,x_1), ..., E_n(P,x_n) \prec E'_n(P,x_n)\}$ for a set of meta-statements. With respect to a set of axioms, A, and a set of meta-statements, RP, for relative plausibility, we define the following model ordering between two models of A, M_1 and M_2 with respect to A and RP:
$M_1 \preceq^{A;RP} M_2$ (We say that M_2 is more plausible than M_1 with respect to A and RP) iff the following conditions are satisfied.

1. M_1 and M_2 have the same domain.
2. Every function constant and predicate constant not in P receives the same interpretation in M_1 and M_2.
3. The following statement is true.
$$\bigwedge_{i=1}^{n} \forall x_i((M_1 \not\models E_i(P,x_i) \wedge M_1 \models E'_i(P,x_i)) \supset (M_2 \not\models E_i(P,x_i) \vee M_2 \models E'_i(P,x_i)))$$
where $RP = \{E_1(P,x_1) \prec E'_1(P,x_1), ..., E_n(P,x_n) \prec E'_n(P,x_n)\}$

The *most plausible models* with respect to A and RP are those models of A, M such that there is no M' such that $M \prec^{A;RP} M'$ (an abbreviated form of $M \preceq^{A;RP} M'$ and not $M' \preceq^{A;RP} M$). We write $MPM(A,RP)$ for a set of all the most plausible models with respect to A and RP.

We explain the idea of the above ordering in a very simple example. Suppose $A = \mathbf{T}$ (no axiom) and $RP = \{P \prec Q\}$. Then the models for A are $M_1 = \{P,Q\}$, $M_2 = \{P,\neg Q\}$, $M_3 = \{\neg P, Q\}$ and $M_4 = \{\neg P, \neg Q\}$ (a model is represented as a set of the propositional constants and negation of propositional constants that are true in the model). By the above ordering, we have only $M_2 \prec^{A;RP} M_3$; that is, if Q is more plausible than P, we can only distinguish possible worlds where P is false and Q is true from other possible worlds where P is true and Q is false.

The reason why we can only distinguish M_2 and M_3 if Q is more plausible than P is as follows. The above models can be regarded as possible worlds for the real world. Therefore, the real world is one of the models M_1, M_2, M_3 and M_4, but we do not know which is actually the real world because of lack of information. Consider the possibilities that a model is the real world, α_i for each model M_i. The possibility that P is true can be expressed as the sum of the possibility of models where P is true. In this case, the possibility of P is $\alpha_1 + \alpha_2$, and the possibility of Q is $\alpha_1 + \alpha_3$. If we know that Q is more plausible than P, then $\alpha_1 + \alpha_2 < \alpha_1 + \alpha_3$; that is, $\alpha_2 < \alpha_3$. Therefore, from the above information on the relative plausibility, we can only conclude that the possibility of M_2 is greater than M_5.

As a result, the most plausible models are M_1, M_3 and M_4. M_1 and M_4 must be considered because we cannot distinguish any model of M_1 and M_4 from any other model. We only conclude that $\neg(P \wedge \neg Q)$ is true in the most plausible models.

From the above definition, we have the following properties for the relative plausibility.

1. If $E(P,x) \prec E'(P,x) \in RP$ and A entails $\forall x(E(P,x) \supset E'(P,x))$, then $MPM(A, RP) = MPM(A, RP - \{E(P,x) \prec E'(P,x)\}$.

   This means that if $\forall x(E(P,x) \supset E'(P,x))$ is true, the information of the relative plausibility is irrelevant. This property corresponds to the property of circumscription; if $\neg P$ is true, then there is no effect of minimizing P.

2. If $E(P,x) \prec E'(P,x) \in RP$ and A entails $\forall x(E'(P,x) \supset E(P,x))$, then $MPM(A, RP) = MPM(A, RP - \{E(P,x) \prec E'(P,x)\}$.

   This means that if $\forall x(E'(P,x) \supset E(P,x))$ is true, the information of the relative plausibility is irrelevant. This property corresponds to the property of circumscription; if P is true, then there is no effect of minimizing P.

3. If $E(P,x) \prec E'(P,x) \in RP$ and $\neg E'(P,x') \prec \neg E(P,x') \in RP$, then $MPM(A, RP) = MPM(A, RP - \{E(P,x) \prec E'(P,x)\}$ and $MPM(A, RP) = MPM(A, RP - \{\neg E'(P,x') \prec \neg E(P,x')\}$.

   This means that an ordering, $E(P,x) \prec E'(P,x) \in RP$, is compatible to an ordering, $\neg E'(P,x') \prec \neg E(P,x')$.

4. If $E(P,x) \prec E'(P,x) \in RP$ and $E'(P,x') \prec E(P,x') \in RP$, then $MPM(A, RP) \subseteq MPM(A, RP - \{E(P,x) \prec E'(P,x), E'(P,x') \prec E(P,x')\})$.

   Usually, if we add more information on the relative plausibility, we can obtain more results in the most plausible models, but if we add contradictory information like the above condition, we obtain fewer results in the most plausible models.

5. Transitivity is not always satisfied. That is, even if $E_1(P,x) \prec E_2(P,x) \in RP$ and $E_2(P,x') \prec E_3(P,x') \in RP$, it is not always:
   $MPM(A, RP) = MPM(A, RP \cup \{E_1(P,x) \prec E_3(P,x)\})$.
   For example, suppose $RP_1 = \{P \prec Q, Q \prec R\}$ and $RP_2 = \{P \prec Q, Q \prec R, P \prec R\}$ and $A = \neg Q$.
   Then $MPM(A, RP_1) = MPM(A, \{\})$ because $Q \supset P$ and $Q \supset R$,
   but $MPM(A, RP_2) = MPM(A, \{P \prec R\}) \neq MPM(A, \{\})$.

   However, transitivity is satisfied if there is no contradictory information to the information of the relative plausibility.

6. If the number of models is finite, then there will be most plausible models. However, if it is infinite, there may not be any most plausible models. This means that the propositional case of the relative plausibility is satisfiable.

3 Syntactic Definition of Relative Plausibility

Syntactically, relative plausibility is defined in a second-order language. In a second-order language, we can use predicate variables and function variables in addition to object variables. Predicate variables vary over predicates and function variables vary over functions. In addition, we use predicate constants such as **T** for true, **F** for false and $=$ for equality, and logical connectives such as $\equiv$ for equivalence.

Let A be a set of axioms and RP be a set of meta-statements of relative plausibility, $\{E_1(P,x_1) \prec E_1'(P,x_1), ..., E_n(P,x_n) \prec E_n'(P,x_n)\}$. We define the most plausible sentence, $MP(A,RP)$, as follows:

$$MP(A,RP) \stackrel{\text{def}}{=}$$
$$A(P) \wedge \neg\exists p(A(p) \wedge$$
$$\bigwedge_{i=1}^{n} \forall x((\neg E_i(P,x) \wedge E_i'(P,x)) \supset (\neg E_i(p,x) \vee E_i'(p,x)))\wedge$$
$$\neg \bigwedge_{i=1}^{n} \forall x((\neg E_i(p,x) \wedge E_i'(p,x)) \supset (\neg E_i(P,x) \vee E_i'(P,x))))$$

where p is a tuple of predicate variables similar to P.

We can easily show that the above definition is an extended form of formula circumscription by substituting $E_i'(P,x)$ and $E_i'(p,x)$ with $\neg E_i(P,x)$ and $\neg E_i(p,x)$ respectively.

We can show that a set of models of $MP(A,RP)$ is identical to $MPM(A,RP)$. This means that any result derived from $MP(A,RP)$ is true in all the most plausible models.

Firstly, we define a model for a second-order language. A *structure*, M, for a second-order language consists of a domain D, which is a non-empty set, and an interpretation function such that every n-ary function constant, F_n, is mapped onto a function from D^n to D (written $M[F_n]$), and every n-ary predicate constant, P_n, is mapped into a subset of D^n (written $M[P_n]$). N-ary function variables range over any function from D^n to D, and n-ary predicate variables range over any subset of D^n. $< t_1, ..., t_n >_M$ denotes an interpreted tuple where $t_1, ..., t_n$ are terms. If $P_n(t_1, ..., t_n)$ is true in M, this fact is expressed as $< t_1, ..., t_n >_M \in M[P_n]$. A *model* of a second-order sentence is any structure, M, such that every formula in the set is true in M. Now we give relationship between the most plausible models and the models of $MP(A,RP)$.

Proposition 1. *M is a model of MP(A,RP) iff M is the most plausible model with respect to A and RP.*

4 Examples

Example 1

Suppose that a symptom of a patient is S_1 and there are the following information of relative plausibility.

- A disease, D_2, is more plausible than a disease, D_1.
- If S_1 is found then it is likely that a patient suffers from D_1 or D_2.

We represent the above information as follows.

$A(\langle S_1, D_1, D_2\rangle) = S_1$

$RP = \{D_1 \prec D_2, S_1 \supset \neg(D_1 \vee D_2) \prec S_1 \supset (D_1 \vee D_2)\}$

Then we can make the following derivation.

$$S_1 \wedge \neg\exists p\exists q\exists r(r \wedge ((\neg D_1 \wedge D_2) \supset (\neg p \vee q))\wedge$$
$$((\neg(S_1 \supset \neg(D_1 \vee D_2)) \wedge (S_1 \supset (D_1 \vee D_2))) \supset (\neg(r \supset \neg(p \vee q)) \vee (r \supset (p \vee q))))\wedge$$
$$\neg($$
$$((\neg p \wedge q) \supset (\neg D_1 \vee D_2))\wedge$$
$$((\neg(r \supset \neg(p \vee q)) \wedge (r \supset (p \vee q))) \supset (\neg(S_1 \supset \neg(D_1 \vee D_2)) \vee (S_1 \supset (D_1 \vee D_2))))))$$

If (p, q, r) is assigned to $(\mathbf{F}, \mathbf{T}, \mathbf{T})$, the above becomes:

$$S_1 \wedge \neg(\mathbf{T} \wedge ((\neg D_1 \wedge D_2) \supset (\neg\mathbf{F} \vee \mathbf{T}))\wedge$$
$$((\neg(S_1 \supset \neg(D_1 \vee D_2)) \wedge (S_1 \supset (D_1 \vee D_2))) \supset (\neg(\mathbf{T} \supset \neg(\mathbf{F} \vee \mathbf{T})) \vee (\mathbf{T} \supset (\mathbf{F} \vee \mathbf{T}))))\wedge$$
$$\neg(((\neg\mathbf{F} \wedge \mathbf{T}) \supset (\neg D_1 \vee D_2))\wedge$$
$$((\neg(\mathbf{T} \supset \neg(\mathbf{F} \vee \mathbf{T})) \wedge (\mathbf{T} \supset (\mathbf{F} \vee \mathbf{T}))) \supset$$
$$(\neg(S_1 \supset \neg(D_1 \vee D_2)) \vee (S_1 \supset (D_1 \vee D_2))))))$$

This is reduced to:

$S_1 \wedge (\neg D_1 \vee D_2) \wedge (\neg S_1 \vee D_1 \vee D_2)$

which is equivalent to:

$S_1 \wedge D_2$.

This means that in the most plausible models, S_1 and D_2 are true.

Example 2

Suppose that a symptom, S_1, is found and S_2 is unknown and there are the following information of relative plausibility.

- If S_1 is found then it is likely that a patient suffers from D_1 or D_2.
- If S_1 is found and $\neg S_2$ is found, then a patient suffers from D_2 more likely than D_1.
- If S_1 and S_2 are both found, then a patient suffers from D_1 more likely than D_2.
- S_2 is rare.

We represent the above information as follows.

$A(\langle S_1, S_2, D_1, D_2\rangle) = S_1$

$RP = \{S_1 \supset \neg(D_1 \vee D_2) \prec^1 S_1 \supset (D_1 \vee D_2), (S_1 \wedge \neg S_2) \supset D_1 \prec (S_1 \wedge \neg S_2) \supset D_2,$
$(S_1 \wedge S_2) \supset D_2 \prec (S_1 \wedge S_2) \supset D_1, S_2 \prec \neg S_2\}$

Then we can make the following derivation.

$$S_1 \wedge \neg\exists p\exists q\exists r\exists s(r\wedge$$
$$((\neg(S_1 \supset \neg(D_1 \vee D_2)) \wedge (S_1 \supset (D_1 \vee D_2))) \supset (\neg(r \supset \neg(p \vee q)) \vee (r \supset (p \vee q))))\wedge$$

$((\neg((S_1 \wedge \neg S_2) \supset D_1) \wedge ((S_1 \wedge \neg S_2) \supset D_2)) \supset (\neg((r \wedge \neg s) \supset p) \vee ((r \wedge \neg s) \supset q)))\wedge$
$((\neg((S_1 \wedge S_2) \supset D_2) \wedge ((S_1 \wedge S_2) \supset D_1)) \supset (\neg((r \wedge s) \supset q) \vee ((r \wedge s) \supset p)))\wedge$
$((\neg S_2 \wedge \neg S_2) \supset (\neg s \vee \neg s))\wedge$
$\neg(((\neg(r \supset \neg(p \vee q)) \wedge (r \supset (p \vee q))) \supset (\neg(S_1 \supset \neg(D_1 \vee D_2)) \vee (S_1 \supset (D_1 \vee D_2))))\wedge$
$((\neg((r \wedge \neg s) \supset p) \wedge ((r \wedge \neg s) \supset q)) \supset (\neg((S_1 \wedge \neg S_2) \supset D_1) \vee ((S_1 \wedge \neg S_2) \supset D_2)))\wedge$
$((\neg((r \wedge s) \supset q) \wedge ((r \wedge s) \supset p)) \supset (\neg((S_1 \wedge S_2) \supset D_2) \vee ((S_1 \wedge S_2) \supset D_1)))\wedge$
$((\neg s \wedge \neg s) \supset (\neg S_2 \vee \neg S_2))))$

If (p, q, r, s) is assigned to $(\mathbf{F}, \mathbf{T}, \mathbf{T}, \mathbf{F})$, the above becomes:

$S_1 \wedge \neg(\mathbf{T}\wedge$
$((\neg(S_1 \supset \neg(D_1 \vee D_2)) \wedge (S_1 \supset (D_1 \vee D_2))) \supset (\neg(\mathbf{T} \supset \neg(\mathbf{F} \vee \mathbf{T})) \vee (\mathbf{T} \supset (\mathbf{F} \vee \mathbf{T}))))\wedge$
$((\neg((S_1 \wedge \neg S_2) \supset D_1) \wedge ((S_1 \wedge \neg S_2) \supset D_2)) \supset (\neg((\mathbf{T} \wedge \neg\mathbf{F}) \supset \mathbf{F}) \vee ((\mathbf{T} \wedge \neg\mathbf{F}) \supset \mathbf{T})))\wedge$
$((\neg((S_1 \wedge S_2) \supset D_2) \wedge ((S_1 \wedge S_2) \supset D_1)) \supset (\neg((\mathbf{T} \wedge \mathbf{F}) \supset \mathbf{T}) \vee ((\mathbf{T} \wedge \mathbf{F}) \supset \mathbf{F})))\wedge$
$((\neg S_2 \wedge \neg S_2) \supset (\neg\mathbf{F} \vee \neg\mathbf{F}))\wedge$
$\neg(((\neg(\mathbf{T} \supset \neg(\mathbf{F} \vee \mathbf{T})) \wedge (\mathbf{T} \supset (\mathbf{F} \vee \mathbf{T}))) \supset$
$(\neg(S_1 \supset \neg(D_1 \vee D_2)) \vee (S_1 \supset (D_1 \vee D_2))))\wedge$
$((\neg((\mathbf{T} \wedge \neg\mathbf{F}) \supset \mathbf{F}) \wedge ((\mathbf{T} \wedge \neg\mathbf{F}) \supset \mathbf{T})) \supset$
$(\neg((S_1 \wedge \neg S_2) \supset D_1) \vee ((S_1 \wedge \neg S_2) \supset D_2)))\wedge$
$((\neg((\mathbf{T} \wedge \mathbf{F}) \supset \mathbf{T}) \wedge ((\mathbf{T} \wedge \mathbf{F}) \supset \mathbf{F})) \supset (\neg((S_1 \wedge S_2) \supset D_2) \vee ((S_1 \wedge S_2) \supset D_1)))\wedge$
$((\neg\mathbf{F} \wedge \neg\mathbf{F}) \supset (\neg S_2 \vee \neg S_2))))$

The above is reduced to:

$S_1 \wedge \neg(\neg($
$(\neg(S_1 \supset \neg(D_1 \vee D_2)) \vee (S_1 \supset (D_1 \vee D_2)))\wedge$
$(\neg((S_1 \wedge \neg S_2) \supset D_1) \vee ((S_1 \wedge \neg S_2) \supset D_2))\wedge$
$(\neg S_2 \vee \neg S_2)))$

which is equivalent to: $S_1 \wedge \neg S_2 \wedge D_2$.

Example 3

Suppose that addition to S_1 in the previous example, a symptom, S_2, is also found. Then we add S_2 to a set of axioms in the previous example.

$A((S_1, S_2, D_1, D_2)) = S_1 \wedge S_2$
$RP = \{S_1 \supset \neg(D_1 \vee D_2) \prec S_1 \supset (D_1 \vee D_2), (S_1 \wedge \neg S_2) \supset D_1 \prec (S_1 \wedge \neg S_2) \supset D_2,$
$(S_1 \wedge S_2) \supset D_2 \prec (S_1 \wedge S_2) \supset D_1, S_2 \prec \neg S_2\}$

Then we can make the following derivation.

$S_1 \wedge S_2 \wedge \neg\exists p \exists q \exists r \exists s(r \wedge s\wedge$
$((\neg(S_1 \supset \neg(D_1 \vee D_2)) \wedge (S_1 \supset (D_1 \vee D_2))) \supset (\neg(r \supset \neg(p \vee q)) \vee (r \supset (p \vee q))))\wedge$
$((\neg((S_1 \wedge \neg S_2) \supset D_1) \wedge ((S_1 \wedge \neg S_2) \supset D_2)) \supset (\neg((r \wedge \neg s) \supset p) \vee ((r \wedge \neg s) \supset q)))\wedge$
$((\neg((S_1 \wedge S_2) \supset D_2) \wedge ((S_1 \wedge S_2) \supset D_1)) \supset (\neg((r \wedge s) \supset q) \vee ((r \wedge s) \supset p)))\wedge$
$((\neg S_2 \wedge \neg S_2) \supset (\neg s \vee \neg s))\wedge$
$\neg(((\neg(r \supset \neg(p \vee q)) \wedge (r \supset (p \vee q))) \supset (\neg(S_1 \supset \neg(D_1 \vee D_2)) \vee (S_1 \supset (D_1 \vee D_2))))\wedge$
$((\neg((r \wedge \neg s) \supset p) \wedge ((r \wedge \neg s) \supset q)) \supset (\neg((S_1 \wedge \neg S_2) \supset D_1) \vee ((S_1 \wedge \neg S_2) \supset D_2)))\wedge$
$((\neg((r \wedge s) \supset q) \wedge ((r \wedge s) \supset p)) \supset (\neg((S_1 \wedge S_2) \supset D_2) \vee ((S_1 \wedge S_2) \supset D_1)))\wedge$
$((\neg s \wedge \neg s) \supset (\neg S_2 \vee \neg S_2))))$

If (p, q, r, s) is assigned to $(\mathbf{T}, \mathbf{F}, \mathbf{T}, \mathbf{T})$, the above becomes:

$S_1 \wedge S_2 \wedge \neg(\mathbf{T} \wedge \mathbf{T}\wedge$

$((\neg(S_1 \supset \neg(D_1 \vee D_2)) \wedge (S_1 \supset (D_1 \vee D_2))) \supset (\neg(\mathbf{T} \supset \neg(\mathbf{T} \vee \mathbf{F})) \vee (\mathbf{T} \supset (\mathbf{T} \vee \mathbf{F}))))\wedge$
$((\neg((S_1 \wedge \neg S_2) \supset D_1) \wedge ((S_1 \wedge \neg S_2) \supset D_2)) \supset (\neg((\mathbf{T} \wedge \neg\mathbf{T}) \supset \mathbf{T}) \vee ((\mathbf{T} \wedge \neg\mathbf{T}) \supset \mathbf{F})))\wedge$
$((\neg((S_1 \wedge S_2) \supset D_2) \wedge ((S_1 \wedge S_2) \supset D_1)) \supset (\neg((\mathbf{T} \wedge \mathbf{T}) \supset \mathbf{F}) \vee ((\mathbf{T} \wedge \mathbf{T}) \supset \mathbf{T})))\wedge$
$((\neg S_2 \wedge \neg S_2) \supset (\neg\mathbf{T} \vee \neg\mathbf{T}))\wedge$
$\neg(((\neg(\mathbf{T} \supset \neg(\mathbf{T} \vee \mathbf{F})) \wedge (\mathbf{T} \supset (\mathbf{T} \vee \mathbf{F}))) \supset$
$(\neg(S_1 \supset \neg(D_1 \vee D_2)) \vee (S_1 \supset (D_1 \vee D_2))))\wedge$
$((\neg((\mathbf{T} \wedge \neg\mathbf{T}) \supset \mathbf{T}) \wedge ((\mathbf{T} \wedge \neg\mathbf{T}) \supset \mathbf{F})) \supset$
$(\neg((S_1 \wedge \neg S_2) \supset D_1) \vee ((S_1 \wedge \neg S_2) \supset D_2)))\wedge$
$((\neg((\mathbf{T} \wedge \mathbf{T}) \supset \mathbf{F}) \wedge ((\mathbf{T} \wedge \mathbf{T}) \supset \mathbf{T})) \supset (\neg((S_1 \wedge S_2) \supset D_2) \vee ((S_1 \wedge S_2) \supset D_1)))\wedge$
$((\neg\mathbf{T} \wedge \neg\mathbf{T}) \supset (\neg S_2 \vee \neg S_2))))$

The above is reduced to:

$S_1 \wedge S_2 \wedge \neg(\neg(\neg S_2)\wedge$
$\neg((\neg(S_1 \supset \neg(D_1 \vee D_2)) \vee (S_1 \supset (D_1 \vee D_2)))\wedge$
$(\neg((S_1 \wedge S_2) \supset D_2) \vee ((S_1 \wedge S_2) \supset D_1))))$

which is equivalent to: $S_1 \wedge S_2 \wedge D_1$

Note that this result is different from the previous result. Therefore, this example shows nonmonotonicity of derivation of the relative plausibility.

Also note that in this derivation, there is an important assumption on the conditional probability. In the probability theory, if the state of known information changes, the probability may change; therefore, the relative probability may also change. On the other hand, in our formalism, the relative plausibility relation is assumed to be invariant, but ignored when the state of known information is contradictory to it. We have a problem if this assumption is not applicable. We show a solution to the problem in the next example.

Example 4

Suppose a disease, D_1, is usually more plausible than a disease, D_2, but if a symptom, S_1, is found then the plausibility relation is reversed. At first glance, it seems that the above relation can be expressed as follows.

$RP = \{D_2 \prec D_1, S_1 \supset D_1 \prec S_1 \supset D_2\}$

If $D_1 \vee D_2$ is true and S_1 is unknown or found to be false, it works. However, if $D_1 \vee D_2$ is true and a symptom, S_1, is found, we cannot conclude D_2.

To solve this problem, we introduce a predicate, Ab, like circumscription. That is,

$RP = \{\neg Ab \supset D_2 \prec \neg Ab \supset D_1, Ab \prec \neg Ab, S_1 \supset D_1 \prec S_1 \supset D_2\}$

and we add the following axiom to A:

$S_1 \supset Ab$

The second plausibility information and the above axiom are needed to derive normal results.

Then if there is no information about S_1, we can conclude D_1, whereas if S_1 is found to be true then we can conclude D_2.

Example 5

Default reasoning is also pos n this framework. Suppose we know that T is a bird and there is informatio f x is a bird then it is plausible that x flies. We can represent the above as fc

$A(\langle B, F\rangle) = B(T)$

$RP = \{B(x) \supset \neg F(x) \prec \quad \supset F(x)\}$

Then we can make the following derivation.

$B(T) \wedge \neg\exists b \exists f(b(T) \wedge$

$\forall x((\neg(B(x) \supset \neg F(x)) \wedge (B(x) \supset F(x))) \supset (\neg(b(x) \supset \neg f(x)) \vee (b(x) \supset f(x)))) \wedge$

$\neg\forall x((\neg(b(x) \supset \neg f(x)) \wedge (b(x) \supset f(x))) \supset (\neg(B(x) \supset \neg F(x)) \vee (B(x) \supset F(x)))))$

The above is equivalent to:

$B(T) \wedge \neg\exists b \exists f(b(T) \wedge$

$\forall x((B(x) \wedge F(x)) \supset (b(x) \supset f(x))) \wedge \neg\forall x((b(x) \wedge f(x)) \supset (B(x) \supset F(x))))$

Let $b = \mathbf{T}$ and $f = \mathbf{T}$. Then from the above formula, we can deduce:

$B(T) \wedge \neg(\mathbf{T} \wedge$

$\forall x((B(x) \wedge F(x)) \supset (\mathbf{T} \supset \mathbf{T})) \wedge \neg\forall x((\mathbf{T} \wedge \mathbf{T}) \supset (B(x) \supset F(x))))$

It is reduced to:

$B(T) \wedge \forall x(B(x) \supset F(x))$

Therefore, we can derive $F(T)$ from the above formula.

5 Related Work

In this section, we restrict ourselves to comparison with the non-numerical approach to uncertainty.

5.1 Prioritized Circumscription

[McCarthy86] introduces priority of minimizing predicates into circumscription. This priority is expressed by ordering over a tuple of predicate constants. This priority is used to choose minimized predicates only when minimization of those predicates conflicts, that is, when only some of the predicates can be minimized. However, if all predicates can be minimized, this ordering is irrelevant. For example, suppose $A = P \vee Q$. Then $Circum(A; P > Q)$ gives $\neg P \wedge Q$. In this case, $P > Q$ is used for minimizing P prior to Q. However, suppose $A' = \mathrm{T}$, then $Circum(A; P > Q)$ gives $\neg P \wedge \neg Q$. Therefore, the ordering of the prioritized circumscription can be regarded as the ordering between two infinitesimal probabilities. However, the technique of prioritized circumscription could be used to express meta-order over our order of the relative plausibility.

5.2 Qualitative Probability as an Intensional Logic

[Gärdenfors75] gives a formalization of qualitative probability to introduce a binary operator $\succeq$ into propositional calculus. The intended meaning of $A \succeq B$ is "A is at least as probable as B". His work is to make the interpretation of the operator $\succeq$

agree with some probability measure. Since his logic, QP, includes $\succeq$ in his language, it can express higher-order probability and infer the ordering itself.

On the other hand, our ordering is expressed as a meta-statement which is translated into the second-order language and we only derive the results which are true in all of the most plausible models. Practically, we believe it is enough to give the most plausible result to make the best decision.

5.3 A Logic to Reason about Likelihood

[Halpern84, Halpern87] give a propositional modal logic which deals with likelihood. Instead of an order of formulas, they introduce the modal operator, **L**, where $\mathbf{L}P$ is read as "P is likely". This idea can be expressed in our formalism as $\neg P \prec P$. However, within their logic, the relative plausibility cannot be expressed. As shown in [Halpern84], their translation of conditional statements threatens contradiction. Then they face the *qualification problem* [McCarthy80] to avoid contradiction. On the other hand, our formalism of the relative plausibility never causes contradiction in the propositional case.

6 Conclusion

This paper presented a formalism of the relative plausibility by extending circumscription. The relative plausibility is represented as a meta-statement of ordering between formulas which is compiled into a second-order language. We think that we need to do the following research:

1. We need to find useful subclasses of meta-statements of the relative plausibility to make the second-order language computable and satisfiable.

2. It would be better if we have more deep relationship between the probability theory and the relative plausibility so that the relative plausibility agrees with some ordering in the probability theory like Gärdenfors's QP.

3. If the orderings are contradictory (for example, $A \prec B$ and $B \prec A$ are in RP), there is no result from contradictory orderings. Pratically, however, we can distinguish those contradictory orderings. To represent priority over such orderings, we would need meta-ordering between them.

4. We must consider more examples to evaluate how finely our formalism treats uncertainty.

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