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The Anonym Problem : A Weak Point of Circumscription on Equality

by

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RESEARCH NOTE

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Abstract

We point out a problem, called the anonym problem, that circumscription is weak in proving inequality, and we clarify the limitation of circumscription in proving (in)equality.

1. Introduction

We, humans, are often faced with a lack of available information in solving problems. In some cases, we infer from imperfect knowledge by complementing it with our common sense knowledge, such as " a bird normally flies", without violating consistency. To capture such reasoning formally, McCarthy proposes a form called circumscription [6,7]. However, unfortunately, circumscription is weak in proving inequality and its weakness yields only unsatisfactory results under circumstances where lack of information regarding (in)equality is essential. We show this with an instance, called the anonym problem. Then we clarify the limitation of circumscription in yielding new facts regarding (in)equality.

2. Anonym Problem

First, we must review circumscription. Circumscription is well known to have various versions. In this paper, we consider the parallel circumscription [4] proposed by Lifschitz, because parallel circumscription is a sufficiently general and powerful version in that most of the other versions, for example, predicate circumscription [6], formula circumscription [7] or prioritized circumscription [4], can be expressed using this formulation [7,4,8]. In this paper, we refer to this parallel circumscription simply as "circumscription".

We basically follow Lifschitz's definitions and notation in [4]. For any first order sentence A(P,Z), where P is a tuple of distinct predicate constants and Z a tuple of function and/or predicate constants disjoint with P, the circumscription of P in A with variable Z is the sentence

$$A(P,Z) \land \neg \exists p,z.(A(p,z) \land p < P),$$

denoted by Circum(A;P;Z). Here, p and z are tuples of variables, and $p<P$ expresses that the extension of each member of p is a subset of the extension of the corresponding member of P, and at least one of them is a proper subset. When we need to minimize wff E(x), we especially allow it to be used in the following way,

$$\mathrm{Circum}(A \land \forall x.(P_0(x) \equiv E(x));P_0;Z),$$

where P_0 is a new predicate which does not occur in A. (This is essentially equivalent to formula circumscription and variable circumscription [8].)

Now, let us consider the following situation.

Situation 1:

A man in a living room hears someone knocking on the front door. He knows that if someone knocks it is normally a man...

Under Situation 1, we expect that he would think that the anonym is a man. In this case, circumscription works as our expectation.

His knowledge can be written as follows:

$A = \mathrm{Knocks(Anonym)} \wedge \forall x.(\mathrm{Knocks}(x) \wedge \neg \mathrm{Ab}(x) \supset \mathrm{Man}(x)).$

We circumscribe predicate Ab, allowing predicate Man to vary, that is,

Circum(A; Ab; Man) =

$A \wedge \neg \exists p,z.(\mathrm{Knocks(Anonym)} \wedge \forall x.(\mathrm{Knocks}(x) \wedge \neg p(x) \supset z(x)) \wedge p < \mathrm{Ab}).$

If we substitute $p = \lambda x.(\mathrm{false})$, $z = \mathrm{Knocks}$, then it yields

$A \wedge \neg(\mathrm{false} < \mathrm{Ab}).$

Since false $\leq$ Ab, so we obtain Ab = false. Therefore, from the circumscription we can obtain

Man(Anonym).

Situation 2 (continuing from Situation 1):

...He opens the door, and finds not a man but a woodpecker, Tweety (Tweety is abnormal). Tweety flies away... (This situation continues, however, if it is interrupted, then circumscription revises the last conclusion and concludes $\neg$Man(Anonym) consistently)...He goes back to the living room and he stays there. He hears someone knocking...

Under Situation 2, although he knows of the existence of an abnormal being, we still expect that he would think that the anonym is a man, because he still knows that a knocker is normally a man. Now we write

$$A' = Knocks(Anonym') \wedge Ab(Anonym) \wedge \forall x.(Knocks(x) \wedge \neg Ab(x) \supset Man(x))$$

(to simplify and clarify the point, Knocks(Anonym) is abandoned).

In this case, does circumscription work as our expectation? By only circumscribing Ab with variable Man, we cannot obtain the expected result. The result is

$Ab = \lambda x.(x = Anonym)$.

Therefore, with respect to Anonym' we can obtain only

$$Anonym' \neq Anonym \supset Man(Anonym').$$

This implies that he cannot conclude that this Anonym' is a man as long as he does not know that Anonym' is not the latter Anonym. In this situation, this result will not be so useful. Staying in his living room, he can never infer that this anonym would be a man, moreover, even if he sees what this anonym is, it will imply to know whether he is a man. (And it seems more absurd for him to remember all abnormal things so exactly that he can distinguish them from whomever he meets.)

Circumscription seems to be weak in proving inequalities as we have shown above. We call such difficulties of circumscription that arise in proving with respect to equalities an "anonym problem". Of course, before we call it a "problem", some problems still have to be clarified. Is it really impossible to deduce the above inequality ($Anonym' \neq Anonym$) from any instances of circumscription? (That is, do there exist some predicates (or formulas) minimized and constants (predicates or functions) allowed to vary with which the inequality is deduced?) In general, what inequality can be deduced from circumscription, and what equality? In the next section, we will clarify these problems.

3. Limitation of Circumscription on Equality

Our interest is to know the ability of circumscription in yielding new facts regarding (in)equality which are not yielded without circumscription. Before exploring the ability generally, we solve the following question: Can circumscription yield no new (in)equality at all? The answer is "No, it cannot". We can show counterexamples to the question, which instance that circumscription yields new facts regarding equality in some cases.

Example 1 (on equality): $A1 \equiv Ab(a) \wedge Ab(b)$. Circumscribing Ab while allowing b to vary, we obtain the result that implies that the only individual satisfies Ab. That is, Circum(A1;Ab;b) yields

$$\forall x.(Ab(x) \equiv x = a).$$

From this, we can obtain a new fact regarding equality,

$$a = b.$$

Example 2 (on inequality): $A2 \equiv Q(a) \wedge \neg Q(b) \wedge R(c)$. We circumscribe $\lambda x.(\neg(R(x) \equiv \neg Q(x)))$ while allowing R and c to vary. For this purpose, we introduce a new unary predicate symbol, P_0, such that $P_0 \equiv \lambda x.(\neg(R(x) \equiv \neg Q(x)))$. That is, we consider $Circum(A2 \wedge \forall x.(P_0(x) \equiv \neg(R(x) \equiv \neg Q(x))); P_0; R, c)$. It yields

$$\forall x. \neg P_0(x),$$

therefore, $\forall x.(R(x) \equiv \neg Q(x))$ and then $\neg Q(c)$ are obtained. Therefore,

$$a \neq b$$

holds.

These examples seem to imply the possibilities that circumscription yields new facts regarding equality that we want; however, it turns out to be negative. Here we show this.

We discuss this problem from a model-theoretic standpoint. We use the following notation.

A as a sentence

Mod(A) as the class of models of A

[M] as the domain of model structure M

M[P] as the extension of P in M

|S| as the cardinal number of a set, S

ω as the least infinite ordinal number

(P;Z)min-Mod(A) as the class of (P;Z)-minimal models of A

To start with, we need to recall the model-theoretic meaning of circumscription. A pre-order on the class of structures "$\leqq^{P;Z}$" was introduced [4]. For any structures M_1,M_2, we write $M_1 \leqq^{P;Z} M_2$ if

1) $[M_1] = [M_2]$

2) $M_1[K] = M_2[K]$ for every constant K not in P,Z;

3) $M_1[P_i] \subseteq M_2[P_i]$ for every P_i in P.

We say that a structure is a *(P;Z)-minimal model of A* if the structure is a minimal model of A with respect to $\leqq^{P;Z}$. Then a model of Circum(A:P;Z) is equivalent to a (P;Z)-minimal model of A [5].

Next, we show an interesting property with respect to such minimal models. Generally, for any sentence A and any tuple of constants P,Z, the existence of the (P;Z)-minimal model of A is not guaranteed. However, if there exists a model of A whose domain consists of finite entities, the existence of the (P;Z)-minimal model is guaranteed for any such A and P,Z.

Lemma:

For any $M \in$ Mod(A), if $|[M]| < \omega$ then

there exists a model $N \in$ (P;Z)min-Mod(A) such that $|[M]| = |[N]|$.

Proof of lemma: Since $|[M]| < \omega$, that is, the number of entities in the domain of model M is finite, tuples of entities which satisfy a property, P_i, in M are finite ($i = 1,\ldots,n$, and let P be $P_1,\ldots,P_n$). If we can make a model, M_1, such that the domain is the same as M, every constant not in P,Z has the same extension as in M but, for some tuples of entities which satisfy P_i in M, in other words, for some elements of the extension of P_i, only these tuples are no longer in the extension of any P_i, then we try to make other models from M_1 in the same way. This process implies making a model, M_i, from M_j such that $M_i <^{P;Z} M_j$. Continuing this process similarly, obviously, we can finitely obtain a model, N, from which we can never continue this process. From the definition of the minimal model, N is a minimal model and $|[M]| = |[N]|$. ■

Now, we clarify a limitation of circumscription on equality.

Theorem 1:

For any $M_0 \in \text{Mod}(A)$ such that $|[M_0]| \leqq |[M]|$ for all $M \in \text{Mod}(A)$, and for any $N_0 \in$ (P;Z)min-Mod(A) such that $|[N_0]| \leqq |[N]|$ for all $N \in$ (P;Z)min-Mod(A),

if $|[M_0]| < \omega$ then $|[M_0]| = |[N_0]|$.

Proof of theorem:

From $|[M_0]| < \omega$, there exists $M_0' \in$ (P;Z)min-Mod(A) such that $|[M_0]| = |[M_0']|$ (using Lemma).

From $|[N_0]| \leqq |[N]|$ for any $N \in$ (P;Z)min-Mod(A), $|[N_0]| \leqq |[M_0']|$.

While, (P;Z)min-Mod(A) $\subseteq$ Mod(A), therefore, from $|[M_0]| \leqq |[M]|$ for all $M \in \text{Mod}(A)$, $|[M_0]| \leqq |[N_0]|$.

Therefore,

$|[M_0]| \leqq |[N_0]| \leq |[M_0']| = |[M_0]|$, so we conclude $|[M_0]| = |[N_0]|$. ■

Theorem 1 states that if the least cardinal number of domains of models is finite, that number is equivalent to the least cardinal number of minimal models. This implies that if there exists a model whose domain consists of finite individuals, circumscription expressed in the form of (1) cannot yield a new fact that varies the least cardinal number of models. Finally, we show that the above theorem is also applicable to the circumscription expressed in the form of (2).

Theorem 2:

Let A, P_0 and E be the same in (2). For any model $M \in Mod(A)$, there exists a model $L \in Mod(A \wedge \forall x.(P_0(x) \equiv E(x)))$ such that $|[M]| = |[L]|$, and vice versa.

Proof of theorem 2:

Let M be a model of A. Then we can get a model L by letting L agree with M on all predicates and functions except P_0 and agree with E on P_0, for P_0 does not occur in A. Of course, $|[M]| = |[L]|$. Conversely, any model L of $A \wedge \forall x.(P_0(x) \equiv E(x))$ is a model of A, therefore it holds.

Theorem 2 guarantees that adding $\forall x.(P_0(x) \equiv E(x))$ to A never causes a change in cardinality of domain of a model. So we conclude from theorem1 and theorem2 that there exists a model whose domain consists of finite individuals, any circumscription expressible by (1) or (2) cannot yield a new fact that varies the least cardinal number of models.

Example 1 (continued): A structure, M, is a model of A1 such that $M[Ab] = \{M[a]\} = \{M[b]\}$. Therefore, the least cardinal number of A1 models is 1. Also, the least cardinal number of Circum(A1; Ab; b) models is 1.

Example 2 (continued): Both the least cardinal number of A2 models and that of Circum($A2 \land \forall x.(P_0(x) \equiv \neg(R(x) \equiv \neg Q(x)))$; P_0; R, c) are 2.

Example 3 (continued from Situation 2): The least cardinal number of A' is 1 because there exists a model of A' whose domain is a singleton. If circumscription yields

Anonym' $\neq$ Anonym,

then the least cardinal number of its models must be greater than (and not equal to) 1, at least. This conclusion violates the theorem. Therefore, circumscription never states that the anonym (Anonym') will not be the former anonym (Anonym).

4. Related Work

Etherington pointed out that circumscription without variable terms cannot conjecture "a $\neq$ b" in the absence of any knowledge, while the default theory can [3]. Our result shows that no circumscription can yield such inequalities (if it can, the least cardinal number varies from 1 to 2). This results in the fact that circumscription cannot subsume default logic.

5. Conclusion, Remarks and Future works

We have discussed the fact that circumscription is weak to prove inequality. Then, how should we solve the anonym problem? At present, we are exploring two approaches to it. One is to make a new version of circumscription which enlarge the power in dealing with each individual constant. The new version may operate in infinite domains as the theorem suggests. The other approach involves a reflective operation which proves unprovable facts from given knowledge. These researches will be reported as soon as they are ready.

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References

[1]: Etherington,D.,Mercer,R. and Reiter,R.: On the adequacy of predicate circumscription for closed-world reasoning, Technical report 84-5, Dept. of Computer Science, Univ. of British Columbia (1984).

[2]: Etherington,D. and Mercer,R.: Domain circumscription: a reevaluation, *Computational Intelligence* Vol 3 (1987) 94-99.

[3] Etherington,D.: Relating Default Logic and Circumscription, *Proc. of Tenth International Joint Conference on Artificial Intelligence*, Milan, Italy (1987) 489-494.

[4]: Lifschitz,V.: Computing circumscription, *Proc. of Ninth International Joint Conference on Artificial Intelligence*, Los Angeles, CA (1985) 121-127.

[5] Lifschitz,V.: On the Satisfiability of Circumscription, *Artificial Intelligence* 28 (1986) 17-27.

[6]: McCarthy,J.: Circumscription - a form of non-monotonic reasoning, *Artificial Intelligence* 13 (1980) 27-39.

[7]: McCarthy,J.: Application of circumscription to formalizing common-sense knowledge, *Artificial Intelligence* 28 (1986) 89-116.

[8]: Perlis,D. and Minker,J.: Completeness Results for Circumscription, *Artificial Intelligence* 28 (1986) 29-42.