A Relevant Logic Approach to Automated Theorem Finding *

(Preliminary Report)

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為学日益，為道日損。
— 老子，道徳経，第四十八章，約 600 B.C.

To attain knowledge, add things every day.
To attain wisdom, remove things every day.
— Lao Tzu, Tao Te Ching, ch.48, about 600 B.C.

Abstract

The problem of automated theorem finding proposed by Wos asks for criteria that an automated reasoning program can use to find new and interesting theorems, in contrast to proving conjectured theorems supplied by the user. This paper discusses the logical basis of automated theorem finding from the viewpoint of relevant logic. The paper points out why classical mathematical logic and/or its various extensions are not suitable logical tools for solving the problem, and shows that paradox-free relevant logics are more hopeful candidates for the purpose. The paper also presents some results of our experiments on automated theorem finding in NBG set theory with EnCal, an entailment calculus system we are developing.

1. Introduction

Reasoning is the process of drawing new and valid conclusions logically from some premises which are known facts and/or assumed hypothesis. Automated reasoning is concerned with the execution of computer programs that assist in solving problems requiring reasoning. Wos 1988 proposed 33 open research problems in automated reasoning [12]. The thirty-first of these problems is the problem of automated theorem finding (ATF for short) which is the main subject we want to investigate in this paper. The question is as follows:

The problem of ATF [11,13] : What properties can be identified to permit an automated reasoning program to find new and interesting theorems, as opposed to proving conjectured theorems?

The problem of ATF, of course, is still open until now [13]. The most important and difficult requirement of the problem is that, in contrast to proving conjectured theorems supplied by the user, it asks for criteria that an automated reasoning program can use to find some theorems in a field that must be evaluated by theorists of the field as new and interesting theorems. The significance of solving the problem is obvious because an automated reasoning program satisfying the requirement can provide great assistance for scientists in various fields.

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On the other hand, it is probably difficult, if not impossible, to find a sentence form in various scientific publications which is more generally used to describe various definitions, propositions, and theorems than the sentence form of "if ... then ... ." A sentence of the form "if ... then ..." is usually called a conditional which states that there exists a conditional and/or causal relationship between "if part" and "then part" of the sentence. Scientists always use conditionals in their descriptions of various definitions, propositions, and theorems to connect a concept, fact, situation or conclusion and its sufficient conditions. Indeed, a major work of almost all, if not all, scientists is to discover some conditional and/or causal relationships between various phenomena, data, and laws in their research areas.

In logic, the notion abstracted from various conditionals is called "entailment." In general, an entailment, for instance, "A entails B" or "if A then B," must concerns two propositions which are called the antecedent and the consequent of that entailment, respectively. The truth and/or validity of an entailment depends not only on the truths of its antecedent and consequent but also more essentially on a necessarily relevant, conditional, and/or causal relation between its antecedent and consequent. As a result, the notion of entailment plays the most essential role in human logical thinking because any reasoning must invoke it. Therefore, it is historically always the most important subject studied in logic and is regarded as the heart of logic [1].

From the viewpoint of logic, there are at least two kinds of entailments. One kind is empirical entailments and the other kind is logical entailments. The truth and/or validity of an empirical entailment is dependent on the contents of its antecedent and consequent. In contrast, the truth and/or validity of a logical entailment depends only on its abstract form but not on the contents of its antecedent and consequent, and therefore, it is considered to be universally true and/or valid. Indeed, the most intrinsic difference between some different logic systems is to regard which entailments as logical entailments [5,6].

This paper discusses the logical basis of ATF from the viewpoint of relevant logic. The paper points out why classical mathematical logic and/or its various extensions are not suitable logical tools for solving the problem of ATF, and shows that paradox-free relevant logics are more hopeful candidates for the purpose. The rest of the paper is organized as follows: Section 2 defines terminology used in this paper. Section 3 discusses the problem that what logic can be used to underlie ATF. Section 4 proposes our relevant logic approach to ATF. Section 5 presents some results of our experiments with EnCal, an entailment calculus tool we are developing. Finally, some concluding remarks are given in Section 6.

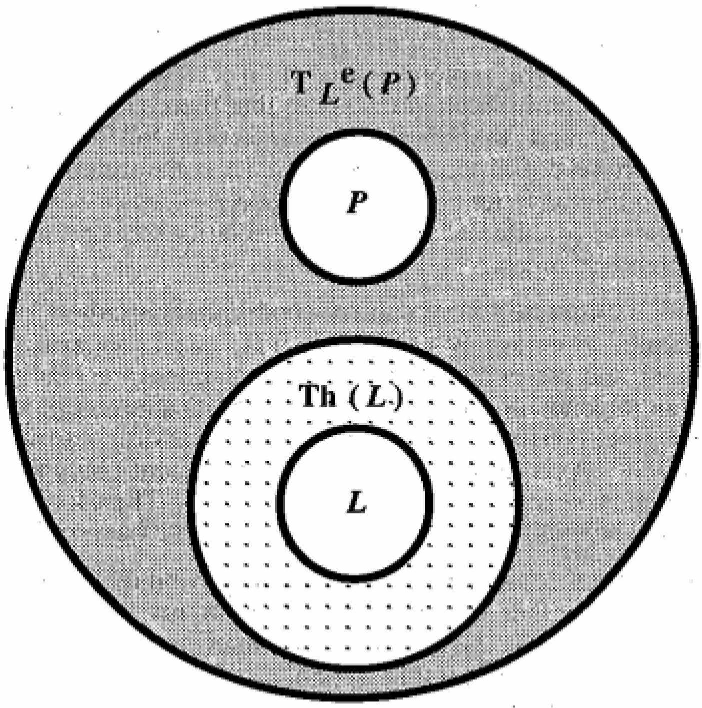
2. Terminology

We now define terminology here for discussing our subject formally.

Definition 2.1 Let L be a logic, "$\vdash_L$" be the proof-theoretical consequence relation of L, Th(L) be the set of all logical theorems of L, and P be a non-empty set of formulas of L. A *formal theory with premises P based on L*, denoted by $T_L(P)$, is defined as follows: $T_L(P) =_{df} Th(L) \cup T_L{}^e(P)$ where $T_L{}^e(P) =_{df} \{A \mid P \vdash_L A \text{ and } A \notin Th(L)\}$ where Th(L) and $T_L{}^e(P)$ is called the *logical part* and the *empirical part* of the formal theory, respectively, and any element of $T_L{}^e(P)$ is called an *empirical theorem* of the formal theory.

Fig. 1 shows the relationship between the logical part and empirical part of a formal theory.

In general, if logic L is adequately strong, then a formal theory $T_L(P)$ based on L is an infinite set of formulas, even though P is a finite set of formulas. Obviously, for any given set of formulas as premises, we can obtain different formal theory based on different logic. However, as we will discuss in Section 3, not all logic systems can serve well as the fundamental logic underlying ATF.



$$T_L(P) =_{df} Th(L) \cup T_L^e(P)$$

Fig. 1 The relationship between the logical part and empirical part of a formal theory

Definition 2.2 A formal theory $T_L(P)$ is said to be *directly inconsistent* if and only if there exists a formula A of L such that both $A \in P$ and $\neg A \in P$ hold. A formal theory $T_L(P)$ is said to be *indirectly inconsistent* if and only if there exists a formula A of L such that any of the following three conditions holds: (1) $A \in P$, $\neg A \notin P$, and $\neg A \in T_L(P)$, (2) $\neg A \in P$, $A \notin P$, and $A \in T_L(P)$, and (3) $A \notin P$, $\neg A \notin P$, $A \in T_L(P)$, and $\neg A \in T_L(P)$. A formal theory $T_L(P)$ is said to be *consistent* if and only if it is neither directly inconsistent nor indirectly inconsistent.

In general, a formal theory constructed as a purely deductive science (e.g., classical mathematical logic and its various extensions) is consistent. However, almost all, if not all, formal theories constructed based on an empirical and/or experiential science is generally indirectly inconsistent.

Definition 2.3 A formal theory $T_L(P)$ is said to be *meaningless* or *explosive* if and only if $A \in T_L(P)$ for arbitrary formula A of L.

Obviously, a meaningless or explosive formal theory is not useful at all.

Now, in our terminology, the problem of ATF can be said as "for any given premises P, how to construct a meaningful formal theory $T_L(P)$ and then find new and interesting theorem in $T_L^e(P)$ automatically?" Since we investigate the problem of ATF from the viewpoint of logic, we have an additional problem as "what logic system can underlie reasoning in ATF?" In the rest of this paper, we want to give primary answers for the problems.

3. On the Logical Basis of ATF

An obvious candidate for the logic to be used to underlie ATF is classical mathematical logic (**CML** for short) where the notion of entailment is represented by the extensional notion of material implication, denoted by $\rightarrow$ here. However, the logic is not a suitable fundamental tool for ATF because of the well-known "implicational paradox problem."

According to the extensional and truth-functional semantics of the material implication, the truth of the formula "A→B" depends only on the truths of A and B, though there could exist no necessarily relevant, conditional, and/or causal relation between A and B. As a result, for example, formulas "snow is white → 1+1=2," "snow is black → 1+1=2," and "snow is black → 1+1=3" are all true in the logic. However, if we read "→" as "if ... then ...," then "if snow is white then 1+1=2," "if snow is black then 1+1=2," and "if snow is black then 1+1=3" are all false in human logical thinking because there is no necessarily relevant, conditional, and/or causal relation between the if-part and the then-part of each sentence. Obviously, in semantics the notion of entailment used in human logical thinking is intrinsically different from the notion of material implication in **CML**. Using the material implication as the entailment is problematical in pragmatics. The "implicational paradox problem" is that if one regards the material implication as the entailment and every logical theorem of **CML** as a valid reasoning form in human logical thinking, then some logical axioms or theorems of the logic, such as "A→(B→A)," "B→(¬A∨A)," "¬A→(A→B)," "(¬A∧A)→B," "(A→B)∨(¬A→B)," "(A→B)∨(A→¬B)," "(A→B)∨(B→A)," "((A∧B)→C)→((A→C)∨(B→C))," and so on, present some paradoxical properties and therefore they have been referred to in the literature as "implicational paradoxes" [1,2,9,10]. For example, in terms of **CML**, formulas "A→(B→A)" and "B→(¬A∨A)" mean "a true proposition is implied by anything"; formulas "¬A→(A→B)" and "(¬A∧A)→B" mean "a false proposition implies anything"; formula "(A→B)∨(B→A)" means "for any two propositions A and B, A implies B or B implies A." However, it is obvious that we cannot say "if B then A" for a true proposition A and an arbitrary proposition B, "if A then B" for a false proposition A and an arbitrary proposition B, and "if A then B or if B then A" for any two irrelevant propositions A and B.

According to Definition 2.1, for any formal theory $T_{CML}(P)$, all implicational paradoxes are logical theorems of $T_{CML}(P)$. As a result, for a conclusion of a deduction from P based on **CML**, we cannot directly accept it as a valid conclusion in the sense of entailment, even if each of given premises P is valid. For example, from any given premise "A", we can infer "B→A," "C→A," ... where B, C, ... are arbitrary formulas, by using logical axiom "A→(B→A)" of **CML** and Modus Ponens for material implication, i.e., $B \to A \in T_{CML}(P)$, $C \to A \in T_{CML}(P)$, ... for any $A \in P \cup T_{CML}(P)$. However, from the viewpoint of human logical thinking, this reasoning is not necessarily regarded as valid in the sense of entailment because there may be no necessarily relevant, conditional, and/or causal relation between B, C, ... and A and therefore we cannot say "if B then A," "if C then A," and so on.

There is another more serious problem as follows if we use **CML** to underlie ATF. Since paradox (A∧¬A)→B is a logical theorem of **CML**, by Modus Ponens, $B \in T_{CML}(P)$ for arbitrary formula B if $T_{CML}(P)$ is directly or indirectly inconsistent. Therefore, if a formal theory $T_{CML}(P)$ is directly or indirectly inconsistent, then it must be meaningless or explosive. This fact shows that **CML** is not a suitable fundamental tool for ATF in empirical and/or experiential sciences because almost all, if not all, formal theories constructed based on an empirical and/or experiential science is generally indirectly inconsistent. This proposition is also true for any of various extensions of **CML** where paradox (A∧¬A)→B is accepted as a logical theorem and Modus Ponens serves as an inference rule.

Note that all of those logic systems (including modal logic systems, intuitionistic logic, and those logic systems developed in recent years for nonmonotonic reasoning) where the entailment is directly or indirectly represented by the material implication have the similar implicational paradox problem as that in **CML**. Therefore, in order to find a right fundamental logic to underlie ATF, we have to investigate some "implicational-paradox-free" logic systems and discuss the validity of reasoning based on them in the sense of the entailment.

Relevant logics are constructed during the 1950s~1970s in order to find a mathematically satisfactory way of grasping the notion of entailment [1,2,9,10]. The first one of such logics is Ackermann's logic system Π'. Ackermann introduced a new primitive connective, called "rigorous implication," which is more natural and stronger than the material implication, and constructed the calculus Π' of rigorous implication which provably avoids those implicational paradoxes. Anderson and Belnap modified and reconstructed Ackermann's system into an equivalent logic system, called "system **E** of entailment." Belnap proposed an implicational relation, called "relevant implication," which is stronger than the material implication but weaker than the rigorous implication, and constructed a calculus called "system **R** of relevant implication." **E** has something like the modality structure of classical modal logic **S4**, and therefore, **E** differs primarily from **R** in that **E** is a system of strict and relevant implication but **R** is a system of only relevant implication. Another important relevant logic system is "system **T** of ticket entailment" or "system **T** of entailment shorn of modality" which is motivated by Anderson and Belnap. A major feature of these relevant logics is that they have a primitive intensional connective to represent the entailment and their logical theorems include no implicational paradoxes [1,2,9,10].

However, although the relevant logics have rejected those implicational paradoxes, there still exist some logical axioms or theorems in the logics which are not natural in the sense of entailment. Such logical axioms or theorems, for instance, are "$(A\wedge B)\Rightarrow A$," "$(A\wedge B)\Rightarrow B$," "$(A\Rightarrow B)\Rightarrow((A\wedge C)\Rightarrow B)$," "$A\Rightarrow(A\vee B)$," "$B\Rightarrow(A\vee B)$," "$(A\Rightarrow B)\Rightarrow(A\Rightarrow(B\vee C))$" and so on, where "$\Rightarrow$" is the primitive intensional connective in the logics to represent the notion of entailment. The present author named these logical axioms or theorems "conjunction-implicational paradoxes" and "disjunction-implicational paradoxes" [4-6]. Similar to the case of **CML**, according to Definition 2.1, for any formal theory $T_{\mathbf{T}}(P)$, $T_{\mathbf{E}}(P)$ or $T_{\mathbf{R}}(P)$, all conjunction-implicational and disjunction-implicational paradoxes are theorems of $T_{\mathbf{T}}(P)$, $T_{\mathbf{E}}(P)$ or $T_{\mathbf{R}}(P)$. As a result, for a conclusion of a deduction from P based on **T**, **E** or **R**, we cannot directly accept it as a valid conclusion in the sense of entailment, even if each of given premises P is valid. For example, from any given premise "$A\Rightarrow B$", we can infer "$(A\wedge C)\Rightarrow B$," "$(A\wedge C\wedge D)\Rightarrow B$," and so on by using logical theorem "$(A\Rightarrow B)\Rightarrow((A\wedge C)\Rightarrow B)$" of **T**, **E** and **R** and Modus Ponens for entailment, i.e., $(A\wedge C)\Rightarrow B\in T_{\mathbf{T/E/R}}(P)$, $(A\wedge C\wedge D)\Rightarrow B\in T_{\mathbf{T/E/R}}(P)$, ... for any $A\Rightarrow B\in P\cup T_{\mathbf{T/E/R}}(P)$. However, from the viewpoint of human logical thinking, this reasoning is not necessarily regarded as valid in the sense of entailment because there may be no necessarily relevant, conditional, and/or causal relation between C, D, ... and B and therefore we cannot say "if A and C then B," "if A and C and D then B," and so on. Therefore, in order to find a right fundamental logic to underlie ATF, we have to investigate some logic systems which are free of not only implicational paradoxes but also conjunction-implicational and disjunction-implicational paradoxes.

Recently, the present author proposed two new relevant logics, named **Ec** and **Rc**, for conditional relation representation and reasoning [7,8]. As a modification of **E** and **R**, **Ec** and **Rc** rejects all conjunction-implicational paradoxes and disjunction-implicational paradoxes in **E** and **R**, respectively, and therefore, they are free of implicational, conjunction-implicational, and disjunction-implicational paradoxes.

Using paradox-free relevant logic systems **Ec** and **Rc** as the fundamental logic to underlie ATF, we can avoid those problems in using **CML**, various extensions of **CML**, and relevant logics **E** and **R**. In the following discussion, we will use **Ec** as our fundamental logic to underlie ATF.

4. ATF by Entailment Calculus

Since a formal theory $T_{\mathbf{Ec}}(P)$ based on **Ec** is generally an infinite set of formulas, even though premises P are finite, we have to find some method to limit the range of candidates for "new and interesting theorems" to a finite set of formulas. The strategy the present author adopted is to sacrifice the completeness of ATF to get the finite set of candidates. This is based on the present author's conjecture that almost all "new and interesting theorems" of a theory can be deduced from the premises of that theory by finite inference steps concerned with finite number of low degree (will be defined below) logical entailments.

Definition 4.1 A formula A is a *zero degree formula* if and only if no entailment connective occurs in it.

Definition 4.2 A formula in the form of $A\Rightarrow B$ is a *first degree formula* (also called a *first degree entailment*) if and only if both A and B are zero degree formulas. A formula in the form of $\neg A$ is a first degree formula if and only if A is a first degree formula. A formula in the form of $A\wedge B$ is a first degree formula if and only if any of the following holds: (1) both A and B are first degree formulas, (2) A is a first degree formula and B is a zero degree formula, and (3) A is a zero degree formula and B is a first degree formula.

Definition 4.3 Let k be a natural number. A formula in the form of $A \Rightarrow B$ is a *kth degree formula* (also called a *kth degree entailment*) if and only if any of the following holds: (1) both A and B are (k–1)th degree formulas, (2) A is a (k–1)th degree formula and B is a jth ($j<k-1$) degree formula, and (3) A is a jth ($j<k-1$) degree formula and B is a (k–1)th degree formula. A formula in the form of $\neg A$ is a kth degree formula if and only if A is a kth degree formula. A formula in the form of $A \wedge B$ is a kth degree formula if and only if any of the following holds: (1) both A and B are kth degree formulas, (2) A is a kth degree formula and B is a jth ($j<k$) degree formula, and(3) A is a jth ($j<k$) degree formula and B is a kth degree formula.

Definition 4.4 Let L be a logic and k be a natural number. A kth degree formula A is a *kth degree logical theorem* of L if and only if $\vdash_L A$.

Definition 4.5 Let L be a logic and k be a natural number. The *kth degree fragment* of L, denoted by L^k, is a set of logical theorems of L such that for any formula A, $A \in L^k$ if and only if (1) A is an axiom of L, or (2) $\vdash_{L^k} A$ and A is a jth ($j \leq k$) degree logical theorem of L.

Note that the kth degree fragment of logic L not necessarily include all kth degree logical theorems of L because it is possible for L that deductions of some kth degree logical theorems of L must invoke those logical theorems whose degrees are higher than k. On the other hand, according to Definition 4.5, the following holds obviously:

$$L^0 \subset L^1 \subset \ldots L^{k-1} \subset L^k \subset L^{k+1} \subset \ldots$$

Definition 4.6 Let L be a logic and k be a natural number. A formula A is said to be *k-deducible from P based on L* if and only if $P \vdash_{L^k} A$ holds but $P \vdash_{L^{k-1}} A$ does not hold.

Note that the notion of k-deducible can be used as a metric to measure the difficulty of deducing an empirical theorem from given premises P based on logic L. The difficulty is relative to the complexity of problem being investigated as well as the strength of underlying logic L.

Based on the above discussion, we have an important result as follows.

Theorem 4.1 Let $T_{\mathbf{Ec}}(P)$ be a formal theory. If P is finite, then all empirical theorems of $T_{\mathbf{Ec}}(P)$ which are k-deducible from P based on $\mathbf{Ec}^k$ must be finite. This is also true even if $T_{\mathbf{Ec}}(P)$ is inconsistent.

Proof Omitted.

Corollary Let $T_{\mathbf{Ec}}(P)$ be a formal theory and k be a natural number. There exists a fixed point P' such that $P \subseteq P'$ and $T_{\mathbf{Ec}k}(P')=P'$. This is also true even if $T_{\mathbf{Ec}}(P)$ is inconsistent.

Proof Omitted.

Note that the proposition that Theorem 4.1 says about relevant logic **Ec** does not hold for those paradoxical logics such as classical mathematical logic **CML** and its various extensions, relevant logics **E** and **R** because these logics accept implicational, conjunction-implicational, or disjunction-implicational paradoxes as logical theorems.

Definition 4.7 Let $T_{\mathbf{Ec}}(P)$ be a formal theory. **Ec** is said to be *kth-degree-complete* for $T_{\mathbf{Ec}}(P)$ if and only if all empirical theorems of $T_{\mathbf{Ec}}{}^e(P)$ are deducible from P based on $\mathbf{Ec}^k$.

Having **Ec** as the fundamental logic and constructing, say the 3rd degree fragment of **Ec** previously, for any given premises P, we can find the fixed point $P'=T_{\mathbf{Ec}3}(P')$. Since the number of 0-deducible, 1-deducible, 2-deducible, and 3-deducible empirical theorems is finite and **Ec** is free of implicational, conjunction-implicational, and disjunction-implicational paradoxes, as a result, we can obtain finite meaningful empirical theorems as candidates for "new and interesting theorems" of formal theory $T_{\mathbf{Ec}}(P)$. Moreover, if **Ec** is 3rd-degree-complete for $T_{\mathbf{Ec}}(P)$, then we can obtain all candidates for "new and interesting theorems" of $T_{\mathbf{Ec}}(P)$. These are also true even if $T_{\mathbf{Ec}}(P)$ is inconsistent. Of course, **Ec** may not be 3rd-degree-complete for $T_{\mathbf{Ec}}(P)$. In this case, a fragment of **Ec** whose degree is higher than 3 must be used if we want to find those 4-deducible empirical theorems, 5-deducible empirical theorems, and so on.

5. Experiments with EnCal

We are developing an entailment calculus system named EnCal which is a general purpose tool for entailment generation and verification. It can generate the kth degree fragment of a specified logic, verify whether a formula is a logic theorem of the kth degree fragment of a specified logic, and generate all k-deducible empirical theorems of a specified formal theory.

Below, we present some current results of our experiments with EnCal.

Table 1 shows a quantitative comparison of logical theorem schemata of 1st, 2nd, and 3rd degree fragments of various logics, where $\mathbf{T}_{\Rightarrow}$, $\mathbf{E}_{\Rightarrow}$, $\mathbf{R}_{\Rightarrow}$, and $\mathbf{CML}_{\rightarrow}$ denotes the purely implicational fragments of relevant logic **T**, **E**, and **R**, and classical mathematical logic **CML** respectively, and $\mathbf{T}_{\Rightarrow,\neg}$, $\mathbf{E}_{\Rightarrow,\neg}$, $\mathbf{R}_{\Rightarrow,\neg}$, and $\mathbf{CML}_{\rightarrow,\neg}$ denotes the implication-negation fragments of **T**, **E**, and **R**, and **CML** respectively. We can see from the table that an enormous number of logical theorems of **CML** are not accepted by relevant logics. This fact also tell us that the consideration to get non-paradoxical logical theorems by filtering paradoxes from **CML** is not practical.

Table 1 A quantitative comparison of logical theorem schemata of various logics

Fragment	$\mathbf{T}_{\Rightarrow}$	$\mathbf{E}_{\Rightarrow}$	$\mathbf{R}_{\Rightarrow}$	$\mathbf{CML}_{\rightarrow}$	$\mathbf{T}_{\Rightarrow,\neg}$	$\mathbf{E}_{\Rightarrow,\neg}$	$\mathbf{R}_{\Rightarrow,\neg}$	$\mathbf{CML}_{\rightarrow,\neg}$
axioms	6	9	7	5	9	12	9	11
1st degree	0	0	0	0	0	0	0	2
2nd degree	0	0	0	4	2	2	0	16
3rd degree	2	2	3	43	70	92	375	40941+

Fragment	**Tc**	**Ec**	**Rc**		**T**	**E**	**R**	**CML**
axioms	12	16	12					
1st degree	0	0	0		∞	∞	∞	∞
2nd degree	7	7	4		∞	∞	∞	∞
3rd degree	630	835	18097		∞	∞	∞	∞

It is well-known that the following relationship holds for three major relevant logics **T**, **E**, and **R**:

$$\mathrm{Th}(\mathbf{T}) \subset \mathrm{Th}(\mathbf{E}) \subset \mathrm{Th}(\mathbf{R})$$

Using EnCal, we have found that the above relationship among fragments of the logics, at lest at 4th degree level, also holds as follows:

$$\mathbf{T}_{\Rightarrow}{}^{3} \subset \mathbf{E}_{\Rightarrow}{}^{3} \subset \mathbf{R}_{\Rightarrow}{}^{3}, \quad \mathbf{T}_{\Rightarrow}{}^{4} \subset \mathbf{E}_{\Rightarrow}{}^{4} \subset \mathbf{R}_{\Rightarrow}{}^{4}$$
$$\mathbf{T}_{\Rightarrow,\neg}{}^{3} \subset \mathbf{E}_{\Rightarrow,\neg}{}^{3} \subset \mathbf{R}_{\Rightarrow,\neg}{}^{3}, \quad \mathbf{T}_{\Rightarrow,\neg}{}^{4} \subset \mathbf{E}_{\Rightarrow,\neg}{}^{4} \subset \mathbf{R}_{\Rightarrow,\neg}{}^{4}$$

At present, it is not know that whether or not the above relationship holds for more higher degree fragments of the logics.

Since almost all mathematics can be formulated in the language of set theory, the set theory has been regarded as the ultimate proving ground for automated theorem proving programs [3,11]. This is also true in ATF. We take set theory as the starting point of our experiments on ATF with EnCal and are finding "new and interesting theorems" in NBG set theory [3,11] by EnCal. The underlying logic we adopted is **Tcqe** which is an extension of **Tc** such that it has quantifier and equality and relative axiom schemata.

Using EnCal, we have found the following: (1) There are 15 1st degree theorems, 46 2nd degree theorems, and 7 3rd degree theorems which are 1-deducible from Gödel's axioms for NBG set theory based on **Tcqe**^{1}. (2) There are 116 1st degree theorems, 9 2nd degree theorems, and 0 3rd degree theorems which are 1-deducible from Quaife's axioms for NBG set theory based on **Tcqe**^{1}. (3) There are 38 1st degree theorems, 186 2nd degree theorems, and 292 3rd degree theorems which are 2-deducible from Gödel's axioms for NBG set theory based on **Tcqe**^{2}. (4) There are 220 1st degree theorems, 324 2nd degree theorems, and 432 3rd degree theorems which are 2-deducible from Quaife's axioms for NBG set theory based on **Tcqe**^{2}. We are continuing the experiment using the 3rd degree fragment of **Tcqe**. We are also doing a comparison of the theorems found by EnCal automatically and the theorems proved by OTTER automatically or semi-automatically.

6. Concluding Remarks

We have pointed out why classical mathematical logic and its various extensions are not suitable logical tools for solving the problem of ATF, and shows that paradox-free relevant logics such as **Tc**, **Ec**, and **Rc** are more hopeful candidates for the purpose. Based on this observation, we have proposed a relevant logic approach to ATF and presented some results of our experiments with EnCal which is a general purpose entailment calculus system.

Although the research presented here is a primary work, it opened a direction for solving the problem of ATF and provided a conceptional foundation for the further research on this direction.

There are many interesting and challenging research problems on the relevant logic approach to ATF presented in this paper. For examples, some important issues are as follows:

(1) Does there exist a decision procedure for the kth-degree-completeness of **Tc**, **Ec**, or **Rc** for any given premises P?

(2) What strategy we should adopt to deal with inconsistency in a formal theory when it is detected in empirical theorem deductions?

(3) How can we define that an empirical theorem is "new" and/or "interesting" formally?

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