When does a First-order Proof Procedure define a good Logic Programming Procedure?

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Abstract

The procedures of the subdiscipline of Disjunctive Logic Programming (DLP) have as their domain of formulas a formula set with the expressive power of the full first-order predicate logic (FOL), the domain of most of the proof procedures of the Automated Theorem Proving (ATP) field. However, there are design considerations that a good proof procedure for DLP should meet that ATP proof procedures need not address. The generic properties are retention of dual readings (declarative and procedural) for programs and a sense of direction for the computation process. These are properties attributable to SLD-resolution and considered by many to be a key reason for the success of Prolog, which is based on SLD-resolution. These properties are implemented by goal-directedness, a positive implication logic, and lack of use of contrapositives of the implicative formulas. We illustrate these points by a comparison of two SLD-resolution extensions, Model Elimination from the ATP community and Near-Horn Prolog, a DLP procedure.

For the major part of the talk we focus on a formal notion of a logic programming language, and subject Near-Horn Prolog to the defined criterion. (We work here with Inheritance Near-Horn Prolog, or InH-Prolog.) We first define an Abstract Logic Programming Language (ALPL) in the sense of Miller and Nadathur. An ALPL is a triple $<\mathcal{D},\mathcal{G},\vdash>$ where $\mathcal{D}$ is the set of program clauses, $\mathcal{G}$ is the set of goal clauses, and $\vdash$ is a proof relation satisfying the *uniform proof* condition. Given finite program $\mathcal{P} \subseteq \mathcal{D}$ and goal formula $G \varepsilon \mathcal{G}$ a proof of sequent

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$\mathcal{P} \longrightarrow G$ is a *uniform proof* (Miller, Nadathur) if and only if it is an intuitionistic proof with a further constraint on the order of introduction of logical symbols in the proof. More specifically, in a Gentzen sequent proof if the current sequent has a non-atomic right-hand side, then the inference rule that created that sequent is a right-introduction rule. After a short introduction to the nature of ALPLs, which includes some interesting relationships between classical, intuitionistic and uniform proof systems in a general setting (results due to Nadathur), we restrict the language under consideration and define an inference system motivated by nH-Prolog. That is, we define an ALPL $< \mathcal{D}, \mathcal{G}, \vdash_{nH} >$ which mimics the InH-Prolog system. We illustrate the construct with a program $\mathcal{P}$ and goal G lacking a uniform proof but where the nH-Prolog modification permits a uniform proof. An interesting side point to this set of results is that as a FOL proof procedure InH-Prolog recognizes as valid some formulas that are only classically valid, yet the process core is a intuitionistically sound process. In fact, it provides another way besides Godel's original result to reflect FOL within the intuitionistic calculus.