Parallel Theorem-Proving System
– MGTP –

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Motivations and Goals

- FOL has a higher expressive power
  than Horn Logic
- TP needs powerful computational resources
  $\rightarrow$ Parallel symbolic processing (KL1/PIM)
- Inefficiency in implementing FOL theorem provers
  $\rightarrow$ Logic programming technologies

Fast General-Purpose Inference Engine on PIM

Why Model Generation ?

- **Problems of KL1 as implementation languages**
  1. **Unsound (unification)**
  2. **Incomplete (search)**

⇩

Model Generation (Forward reasoning)

1. **When dealing with range-restricted clauses, matching is sufficient (KL1's head unification)**
2. **OR parallel searching can be implemented by KL1's AND parallel execution**

Model Generation Method

- **Tries to construct models (atom sets) satisfying a given clause set by forward reasoning**

MG Clauses

Initial mode

$true \rightarrow p(a); r(a).$

Extension rule

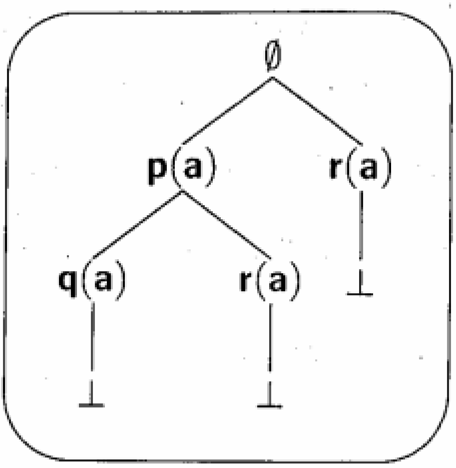
$p(\mathbf{X}) \rightarrow q(\mathbf{X}); r(\mathbf{X}).$

Rejection rule

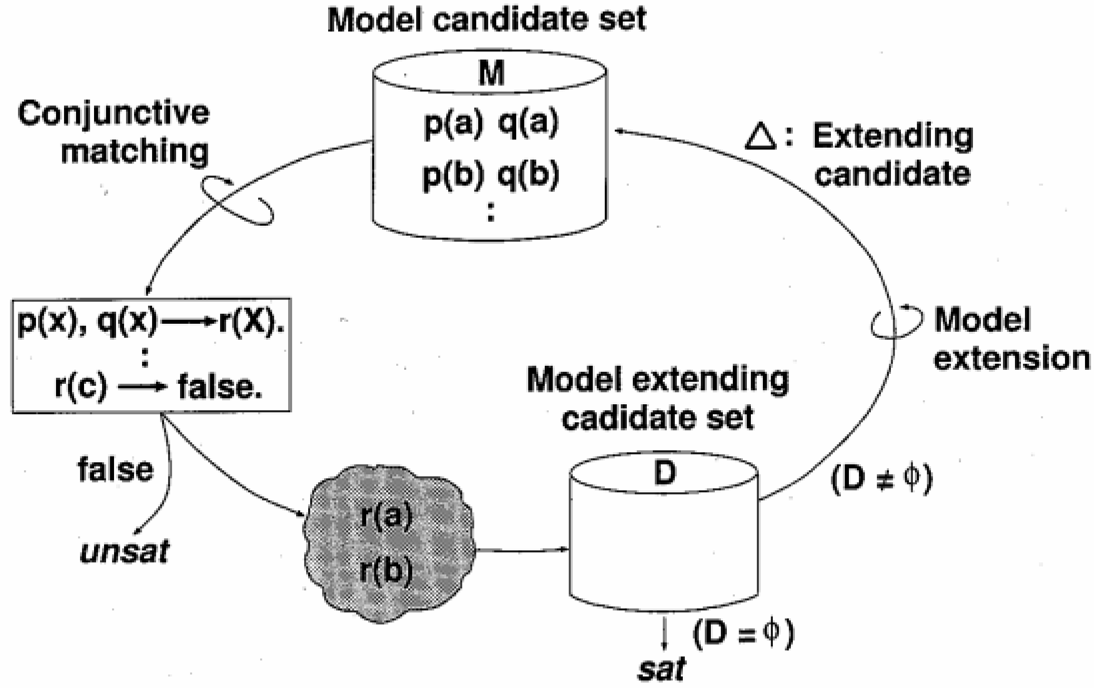
$r(\mathbf{X}) \rightarrow false.$

$p(\mathbf{X}), q(\mathbf{X}) \rightarrow false.$

Proof Tree

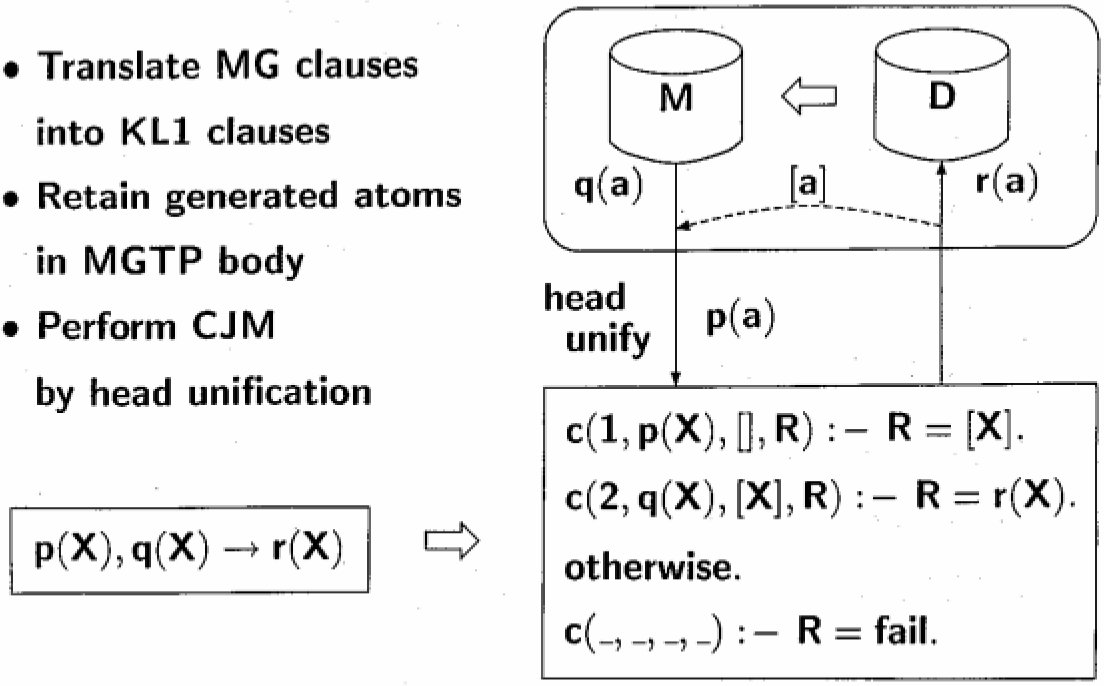


Model Generation Process

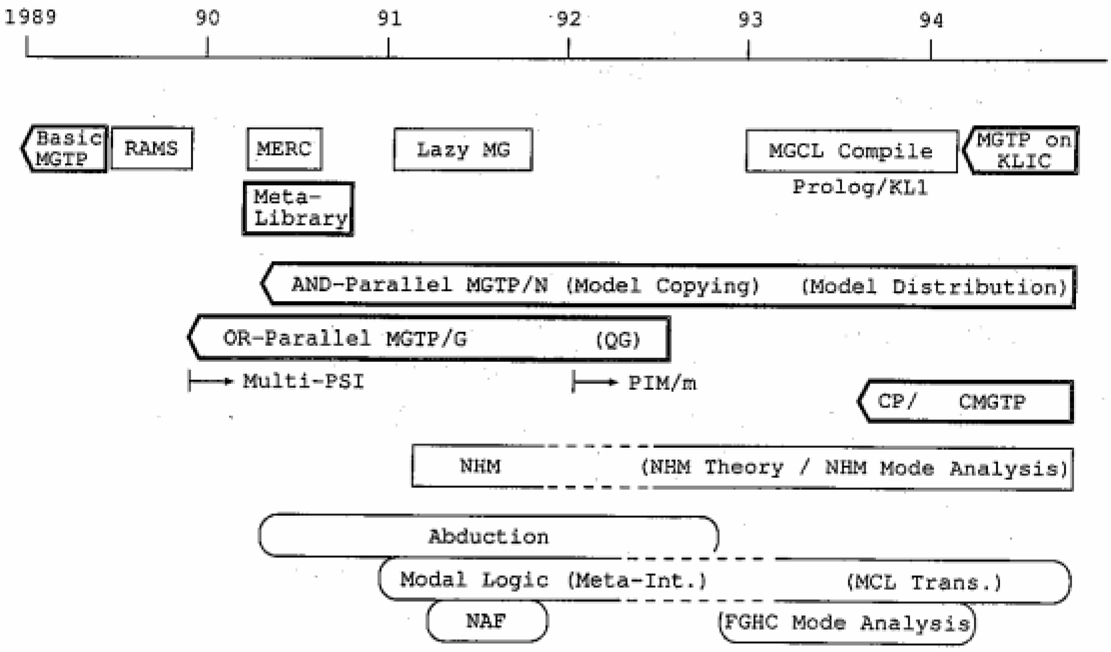


KL1 Implementation of MGTP

- Translate MG clauses into KL1 clauses
- Retain generated atoms in MGTP body
- Perform CJM by head unification

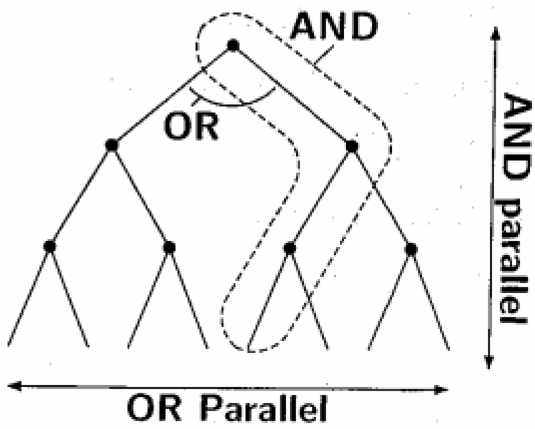


History of MGTP Development

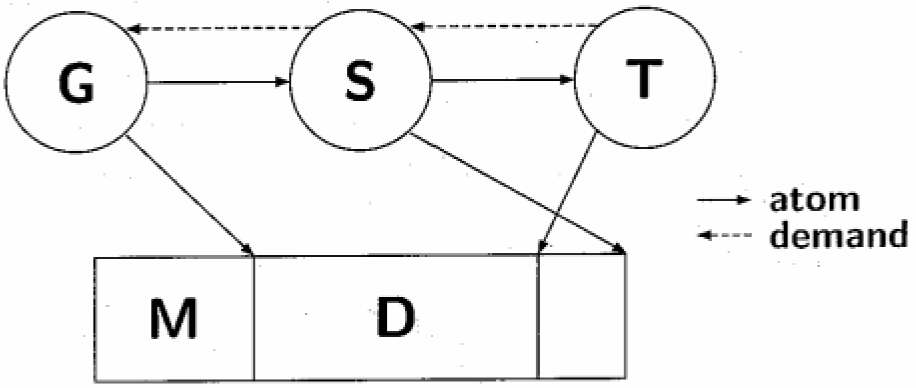


Parallelization of MGTP

- OR Parallelization
  - Explore each branch in parallel
- AND Parallelization
  - Exploit parallelism in searching one branch
    - CJM, Subsumption tests –

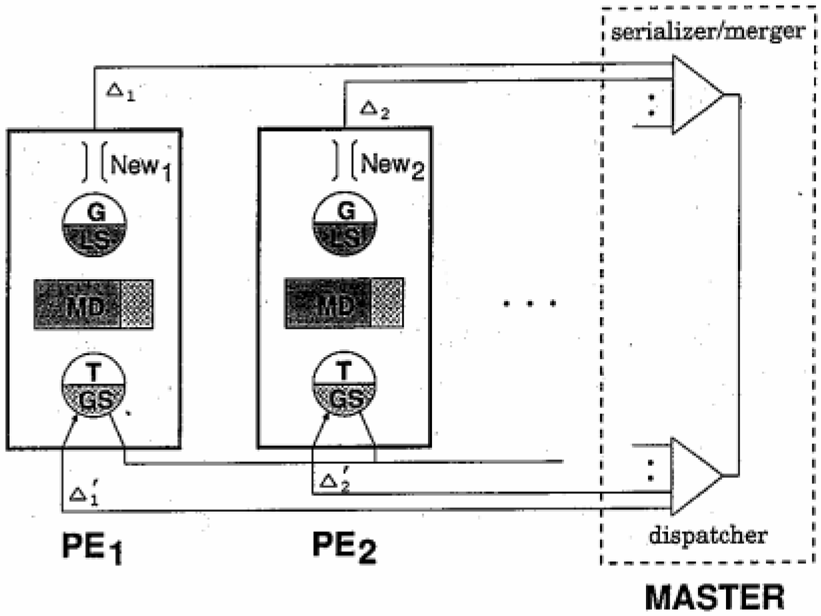


Basic Structure of MGTP Processes



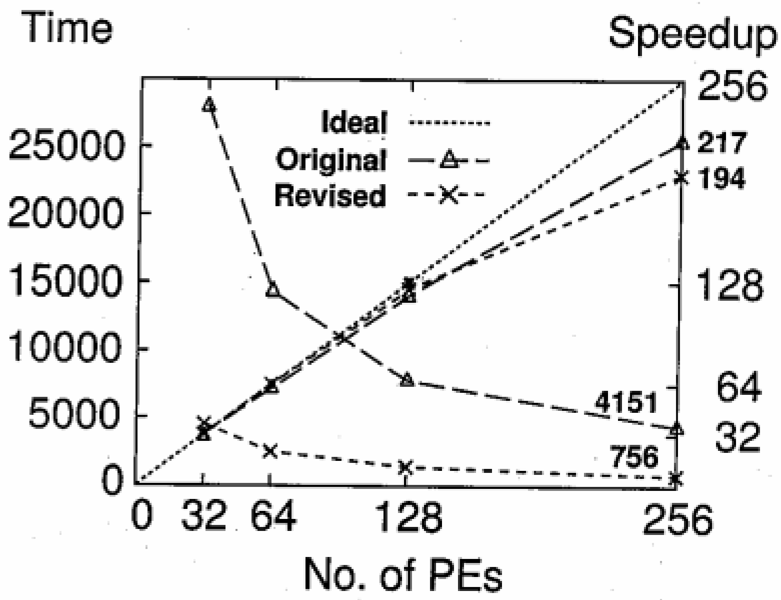
- G process generates new atoms only when requested by T process (Lazy Model Generation)

AND Parallelization
(Model Copying)



AND Parallel Performance on PIM/m

(Av. of 12 condensed detachment problems)



Results of ALL-FAIL Problems

(Condensed detachment)

Prob#	W	|M+D|	Time(sec)	PEs
15	16	9838	244	128
22	16	36497	1590	128
23	16	85100	11047	256
40	16	10024	572	256
43	16	6875	320	256
44	18	15071	1725	256
49	18	20623	2854	256

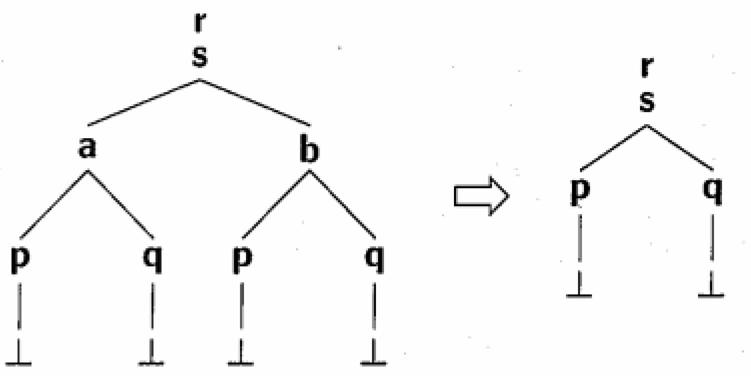
unsolved : 24,34,35,50,55

Extension of MGTP Features

- Non-Horn Magic Set (NHM)
  - Combining bottom-up with top-down
    - To simulate top-down execution by using goal information
    - To avoid generating useless model candidates
- Constraint MGTP (CMGTP)
  - Incorporating negative constraint propagation facilities
  - To enhance MGTP features for finite-domain constraint satisfaction problems

Non-Horn Magic Set (BF-NHM)

$\rightarrow r. \quad \rightarrow s.$			$g(r) \rightarrow r. \quad g(s) \rightarrow s.$
$r, s \rightarrow a; b.$		$g(a), g(b) \rightarrow g(r), g(s).$	$g(a), g(b), r, s \rightarrow a; b.$
$r, s \rightarrow p; q.$	⇨	$g(p), g(q) \rightarrow g(r), g(s).$	$g(p), g(q), r, s \rightarrow p; q.$
$p \rightarrow .$		$\rightarrow g(p).$	$p \rightarrow .$
$q \rightarrow .$		$\rightarrow g(q).$	$q \rightarrow .$



Preserving Range-restrictedness

TD simulation part (N.R.R.)

$\mathbf{q}(\mathbf{c}, \mathbf{X}) \rightarrow .$
$\mathbf{p}(\mathbf{X}, \mathbf{Y}, \mathbf{Z}) \rightarrow \mathbf{q}(\mathbf{X}, \mathbf{Z}).$

$$\rightarrow g(\mathbf{q}(\mathbf{c}, \mathbf{X})).$$
$$g(\mathbf{q}(\mathbf{X}, \mathbf{Z})) \rightarrow g(\mathbf{p}(\mathbf{X}, \mathbf{Y}, \mathbf{Z})).$$

⇩ Using Adornments

$$\rightarrow g(\mathbf{q}^{\mathbf{bf}}(\mathbf{c})).$$
$$g(\mathbf{q}^{\mathbf{bf}}(\mathbf{X})) \rightarrow g(\mathbf{p}^{\mathbf{bff}}(\mathbf{X})).$$
$$\mathbf{q}(\mathbf{c}, \mathbf{X}) \rightarrow .$$
$$g(\mathbf{q}^{\mathbf{bf}}(\mathbf{X})), \mathbf{p}(\mathbf{X}, \mathbf{Y}, \mathbf{Z}) \rightarrow \mathbf{q}(\mathbf{X}, \mathbf{Z}).$$

Evaluation of NHM

Problem	Original		NHM	
	Branches	Time (ms)	Branches	Time (ms)
Ex1	10	13	2	9
Ex2	26,873,856	T.O.	3	12
Ex3	-	T.O.	2	55
Ex4	-	T.O.	9	621
SYN009-1	-	T.O.	3	10
PUZ012-1	4,491	33,610	9	340

† on Pseudo-MultiPSI
Ex1,Ex2 : [Wilson and Loveland 89]
Ex3 : [Reboh et.al. SRI-TR 72]
Ex4 : [Loveland et.al. JACM 74]
†† on SPARC 10/30
SYN009-1, PUZ012-1 : [TPTP Library]

Current Activities on NHM

- **Completeness/Soundness theorems**

- **Correspondences between relevancy testing† and NHM**

  † **SATCHMORE by Loveland et.al.**

- **Mode analysis of necessary adorned predicates for NHM transformation**

Quasigroup Existence Problems (Bennett)

- **Quasigroup** $\langle Q, \circ \rangle$ (Q:set, $\circ$: binary operation on Q)
  $\Longleftrightarrow \forall xyz.(x \circ y \in Q, y \neq z \Rightarrow (x \circ y \neq x \circ z) \wedge (y \circ x \neq z \circ x))$

- **Quasigroup Existence Problems :** Existence problems of latin squares which satisfies some additional constraints :QG.1 $\sim$ QG.7

$\circ$	1	2	3	4	5
1	1	3	2	5	4
2	5	2	4	3	1
3	4	5	3	1	2
4	2	1	5	4	3
5	3	4	1	2	5

An idempotent latin square of order 5 which satisfies QG5 : $yxyy = x$.

CMGTP

- **Negative atoms** can be used to represent negative constraint propagation,
- Extended MGTP rules, such as $P, \neg R \rightarrow \neg Q$ and $\neg R, Q \rightarrow \neg P$ are added to the original rule $P, Q \rightarrow R$,
- **Unit refutation** $A, \neg A \rightarrow false$ is introduced as an integrity constraint.
- **Unit simplification** is performed between atoms in M and disjunctions in D.

$$\frac{\begin{matrix}(M) & (M)\\ \vdots & \vdots \\ \neg A & A\end{matrix}}{false} \qquad \frac{\begin{matrix}(M) & (D)\\ \vdots & \vdots \\ \neg A & \mathcal{D}_1 \vee A \vee \mathcal{D}_2\end{matrix}}{\mathcal{D}_1 \vee \mathcal{D}_2} \qquad \frac{\begin{matrix}(M) & (D)\\ \vdots & \vdots \\ A & \mathcal{D}_1 \vee \neg A \vee \mathcal{D}_2\end{matrix}}{\mathcal{D}_1 \vee \mathcal{D}_2}$$

CMGTP can be considered as a meta language to describe constraint propagation rules directly.

Problem Description in MGTP

QG5: $yxyy = x \Leftrightarrow$ **YX=A, AY=B, BY=C → C=X**

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true → dom(1), dom(2), dom(3), dom(4), dom(5).
dom(M), dom(N) →
   p(M, N, 1); p(M, N, 2); p(M, N, 3); p(M, N, 4); p(M, N, 5).
p(Y, X, V1), p(V1, Y, V2), p(V2, Y, V), {V\= X} → false.
p(X, X, V), {V\= X} → false.
p(X, Y1, V), p(X, Y2, V), {Y1\= Y2} → false.
p(X1, Y, V), p(X2, Y, V), {X1\= X2} → false.
p(X, 5, Y), {X1 is X − 1, Y < X1} → false.
```

Problem Description in CMGTP

MGTP description :

$$p(Y,X,V1), p(V1,Y,V2), p(V2,Y,V), \{V \backslash = X\} \rightarrow false.$$

$$\Downarrow$$

CMGTP description :

$$p(Y,X,V1), p(V1,Y,V2) \rightarrow p(V2,Y,X).$$
$$p(Y,X,V1), not(p(V2,Y,X)) \rightarrow not(p(V1,Y,V2)).$$
$$not(p(V2,Y,X)), p(V1,Y,V2) \rightarrow not(p(Y,X,V1)).$$

Disjunctions are simplified with negative atoms.

Experimental Results on CP and CMGTP

Problem	$\|M\|$	DDPP	FINDER	MGTP	CP	CMGTP
QG5. 9	0	15	40	239	15	15
QG5. 10	0	50	356	7026	38	38
QG5. 11	5	136	1845	51904	117	117
QG5. 12	0	443	13527	2749676	372	372
QG5. 13	0				13924	13924
QG5. 14	0				64541	64541
QG5. 15	0				151250	151250
QG5. 16	0				19382469	

DDPP: Davis&Patnum on SPARC-2, MGTP: on PIM/m-256
FINDER: Finite domain solver on SPARC-10, CP: on SPARC-10

Research on MGTP Applications

– MGTP as a meta-programming language –

We can implement various inference systems on MGTP, by representing the necessary inference rules with MG clauses.

Negation As Failure	Abduction	Modal Logic	Mode Analysis

Negation As Failure

Translate formula with nonmonotonic properties to MG clauses with modality

$$\boxed{A \;:-\; B, \text{not } C} \Rightarrow \boxed{B \rightarrow \neg KC, A \;;\; KC}$$

Integrity Constraints

$$\neg KA, A \rightarrow \text{false}.$$

$$\neg KA, KA \rightarrow \text{false}.$$

Stability at fixpoint

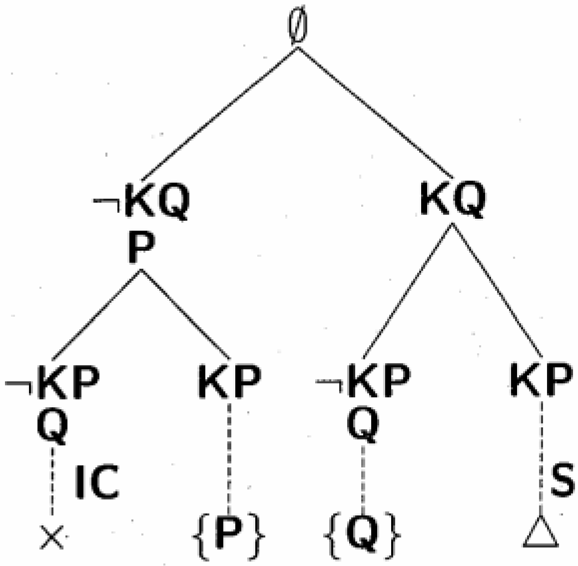
$$\text{if} \quad KA \in M \quad \text{then} \quad A \in M$$

Example (NAF)

P :− not Q
Q :− not P

⇩

true → ¬KQ, P ; KQ
true → ¬KP, Q ; KP



Modal Tableaux on MGTP

(Meta-programming method)

α rule: $t(P \wedge Q, W) \rightarrow t(P, W), t(Q, W)$

β rule: $t(P \vee Q, W) \rightarrow t(P, W); t(Q, W)$

π rule: $f(\Box P, W) \rightarrow \{new\_world(V)\}, path(W, V), f(P, V)$

ν rule: $t(\Box P, W), path(W, V) \rightarrow t(P, V)$

close condition: $t(P, W), f(P, W) \rightarrow false$

t(P,W) : P is true in W   f(P,W) : P is false in W
path(W,V) : V is accessible from W
new_world(W) : create a new world W

Performance of Meta-Programming Method
(PTL)

	MGTP(ms)	ALS(ms)
$\Diamond\Box a \supset \Box\Diamond\Box a$	900	6383
$a \supset \Diamond a$	<1	500
$\bigcirc a \supset \Diamond a$	19	1783
$\Diamond\neg a \equiv \neg\Box a$	10	1583
$\Box(a \wedge b) \equiv (\Box a) \wedge (\Box b)$	50	10250
$\bigcirc\Box a \equiv \Box\bigcirc a$	329	8800
$\Box\Diamond\Box a \equiv \Diamond\Box a$	899	46983
$(a \wedge \Diamond\neg a) \supset \Diamond(a \wedge \bigcirc\neg a)$	239	117167

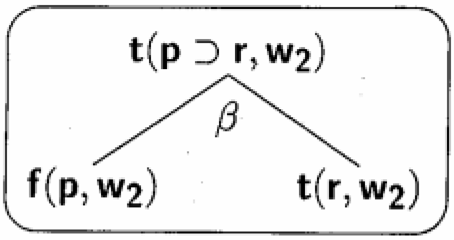
MGTP: SICStus Prolog on SS10/30
ALS: ALS Prolog on SUN3/60M

Modal Clause Transformation

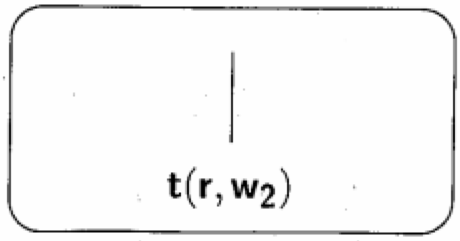
– Partial evaluation of modal rules –

ex. $\underbrace{\Box(p \supset r)}_{\nu} \wedge \Box p \supset \Box r$

$$\nu : t(\Box F, W), path(W, V), \rightarrow t(F, V) \quad \Rightarrow \quad t(\Box(p \supset r), W), path(W, V), t(p, V) \rightarrow t(r, V)$$

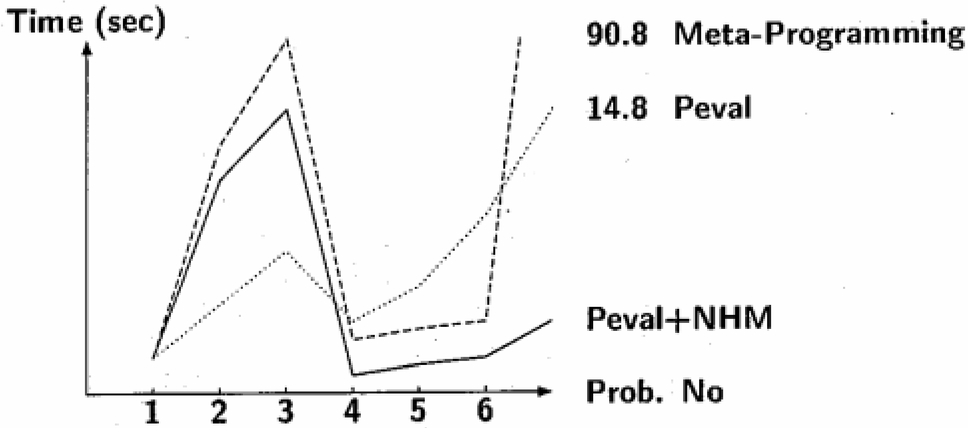


(Meta programming)



(Peval)

Evaluation of Transformation Method



1. 3 wise men 2. A formula whose antecedent contains many literals
3. A formula which produces long inference chains
4. A formula which may produce irrelevant branches
5. A formula which may produce irrelevant worlds
6. 2 & 4

Mode Analysis of FGHC Programs

• Mode propagation	$\Leftrightarrow$	Model extension
• Mode conflict	$\Leftrightarrow$	Model rejection

FGHC clause:

$$\mathsf{app([A|X],Y,Z)\ :-\ true\ |\ Z=[A|Z_1],app(X,Y,Z_1)}$$

MG clause:

$$\mathsf{true} \rightarrow \mathsf{m([\langle app,1\rangle],in),m([\langle app,3\rangle],out)}.$$

$$\mathsf{m([\langle app,1\rangle,\langle .,1\rangle|P],M)} \leftrightarrow \mathsf{m([\langle app,3\rangle,\langle .,1\rangle|P],\overline{M})}.$$

$$\mathsf{m([\langle app,1\rangle,\langle .,2\rangle|P],M)} \leftrightarrow \mathsf{m([\langle app,1\rangle|P],M)}.$$

$$\mathsf{m([\langle app,3\rangle,\langle .,2\rangle|P],M)} \leftrightarrow \mathsf{m([\langle app,3\rangle|P],M)}.$$

$\overline{\mathsf{in}} = \mathsf{out} \quad \overline{\mathsf{out}} = \mathsf{in}$

Performance of MGTP Mode-Analyzer

benchmark	CPU time (msec)			
	prepro I	prepro II	execution	total
msort	600 (6.3%)	8,750 (93.1%)	50 (0.5%)	9,400
queens	920 (7.0%)	12,170 (92.3%)	100 (0.8%)	13,190
cubes	1,560 (7.0%)	19,430 (86.8%)	1,400 (6.3%)	22,390
pascal	1,150 (7.4%)	13,840 (89.2%)	530 (3.4%)	15,520
mandel	2,700 (7.1%)	34,410 (90.8%)	800 (2.1%)	37,910
rucs	1,410 (8.6%)	14,770 (90.6%)	130 (0.8%)	16,310
bestpath	4,340 (6.3%)	52,080 (76.0%)	12,140 (17.7%)	68,560
waltz	3,670 (8.4%)	38,140 (86.8%)	2,120 (4.8%)	43,930
waves	4,670 (7.7%)	51,940 (85.2%)	4,330 (7.1%)	60,940
triangle	14,060 (8.4%)	149,060 (89.3%)	3,810 (2.3%)	166,930
arith mean	(7.7%)	(86.7%)	(5.6%)	

PDSS on SS10/30

Conclusion

- Almost linear speedup was attained with the PIM/m-256 system.

  We succeeded in solving some hard mathematical problems such as QG.

  ⇐ Effectiveness of large-scale parallel TPs
- MGTP can cover a wide class of AI applications.

  Various inference systems can be built on MGTP by writing the inference rules with MG clauses.

  ⇐ MGTP as a meta-programming language

  ⇩

  New Bottom-up Logic Programming Paradigms/Languages