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Logical Implementation of Dynamical Models

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Abstract

In this paper, we explore the logical system which reflect the dynamical model. First, we define the "causality" which requires "time reference". Then, we map the causation to the specific type of logical implications which requires the time fragment $dt > 0$ at each step when causal changes are made. We also propose a set of axioms, which reflect the feature of state-space and the relation between time and state-space. With these axioms and logical implications mapped from the dynamical systems, the dynamical state transition can be deduced logically. We also discussed an alternative way of deducing the dynamical state change using time operators and state-space operators.

1 Introduction

Although the dynamical systems and logical systems are considered to be completely different systems, there are several elements in common. We mapped from dynamical systems to logical systems to investigate the following questions:

(1) How the fundamental concepts in dynamical systems such as observability, stability can be related to those in logical systems such as completeness, soundness ? (2) In order to attain the dynamical simulation on the mapped logical systems, what are necessary ? (3) Can the qualitative simulation be carried out by deducing the future state from the current state and some axioms characterizing time, state-space and their relations ?

We consider it is crucial to discriminate (physical) causality explicitly from logical deducibility. We studied a causality characterized by "the time reference" other than event dependency for the discussion of physical causality. The physical causality (or equivalently "change" through physical time) is intrinsically embedded in a dynamical model which states the causal relation between what is changed and what makes the change.

In this paper, we treat the physical causality as specific type of deduction which always requires the fact of the time fragment $dt > 0$ at each step. By mapping the dynamical model as well as some meta-rules which reflects that the state-space of dynamical systems is continuous to logical rules, the qualitative reasoning on dynamica systems can be done by logical deductions.

Section 2 discusses the causality on the dynamica model. The causality is defined in terms of physica time. Then the causation is mapped to the logical implication which requires time fragment ($dt > 0$) at each step. Cause-effect sequence is obtained by the deduction where the new fact $dt > 0$ is required at each step. Section 3 discusses the relation between some concepts on dynamical models and those on logical systems. Section 4 presents a set of axioms from which state transitions are deduced logically. Section 5 discusses an alternative formalization of logical systems for deducing the dynamical changes.

2 Mapping Causality in Dynamical Models to Logical Implications

2.1 Causality referring to time

The causality has the following two requirements, which seem intuitively sound for a causality for the discussion of dynamical change. When we say "the event A caused the event B", we must admit

(1) Time Reference : The event A occurred "before" the event B, (2) Event Dependency: The occurrence o the event B must be "dependent on" the occurrence o the event A.

The "time reference" plays a crucial role to make clear distinction between "the causality" and logical deduction. In the original dynamical model of the form: $dY/dt = X$

contains the "built-in causal" direction from the right hand side to the left hand side. We restrict ourselves to interpret the form $dY/dt = X$ as follows: $X > 0$ caused $dt > 0$) or is capable of causing the event of Y increase $dY > 0$). The requirement of the new fact $dt > 0$ should be claimed to verify the "built-in causality". Thus the form will be mapped to the logical form:

2.2 Language for dynamics

In order to logically describe the constraint of dynamical model, we use the following *First Order Predicate Calculus.* We use the 4-place predicates p(x,i), n(x,i), z(x,i) which should be interpreted as positive, negative, zero of the variable x at certain moment i. p(x,i), for example is interpreted as follows:

$$p(x,i) = \begin{cases} true, & \text{if x (at time i)} > 0; \\ false, & \text{otherwise.} \end{cases}$$

Since the state must be unique at any moment, these predicates must satisfy the following uniqueness axioms U.

U-(1) $\forall x \forall i (p(x,i) \rightarrow ((\sim n(x,i)) \wedge (\sim z(x,i))))$
U-(2) $\forall x \forall i (n(x,i) \rightarrow ((\sim p(x,i)) \wedge (\sim z(x,i))))$
U-(3) $\forall x \forall i (z(x,i) \rightarrow ((\sim n(x,i)) \wedge (\sim p(x,i))))$

We also use the 2-place predicate of inequality $>(x,y)$. Other than these three predicates, we also use functions such as d/dt(time derivative), $+$(addition), $-$(substraction), $\cdot$(multiplication), /(division) defined on the time varying function x(t) in our language.

With these predicates, the causality defined from X to Y can be written by:

$$p(X(t),i) \wedge p(dt,i) \rightarrow p(dY/dt,i)$$

$$n(X(t),i) \wedge p(dt,i) \rightarrow n(dY/dt,i)$$

$$z(X(t),i) \vee z(dt,i) \rightarrow z(dY/dt,i)$$

2.3 Causality in dynamical models

We formalize the "causality" by the propagation of sign in the dynamical model. In the propagation, time reference is included, since $p(dt,i)$ is always needed to conclude the causation.

Example 2.1.

In order to compare the simulation results with those done by other qualitative simulation [de Kleer and Brown 1984], we use the same example of pressure regulator as shown in Fig. 1.

We can identify the causality in the feedback path. The flow also is caused by a driving force and by the available area for the flow. Further, the pressure at a point is caused by the flow through the point. Reflecting these causal path, the following model is obtained.

$$d\delta Xs/dt = -a \cdot \delta Po - d \cdot \delta Xs$$
$$d\delta Q/dt = b \cdot (\delta Pi - \delta Po - c \cdot (2Xs \cdot Q\delta Q - Q^2 \delta Xs)/Xs^2$$
$$d\delta Po/dt = e \cdot (2Q\delta Q - f \cdot \delta Po)$$

where $a, b, c, d, e,$ and f are appropriately chosen positive constants. δ x denotes the variance from the equilibrium point of x.

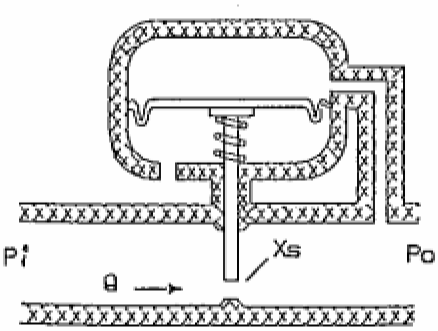

Fig.1 A Schematic Diagram of Pressure Regulator

The first equation of the model, for example, is mapped to the logical formulae:

$$n(\delta po(t),i) \wedge p(dt,i) \rightarrow p(d\delta xs/dt,i)$$

$$p(\delta po(t),i) \wedge p(dt,i) \rightarrow n(d\delta xs/dt,i)$$

$$z(\delta po(t),i) \vee z(dt,i) \rightarrow z(d\delta xs/dt,i)$$

With the set of logical formulae, which are mapped from the dynamical equations and the following axioms, we can obtain a cause-effect sequence by the causal deduction on this model.

I-(1)

$$\forall x \forall i \forall j (z(x,i) \wedge p(dx/dt,i) \wedge (j > i) \rightarrow$$

$\exists k((j > k) \wedge (k > i) \wedge p(x,j,k)))$
I-(2)

$$\forall x \forall i \forall j (z(x,i) \wedge n(dx/dt,i) \wedge (j > i) \rightarrow$$

$\exists k((j > k) \wedge (k > i) \wedge n(x,j,k)))$

These are the instant change rules [de Kleer and Bobrow 1984], which state that $z(x,i)$ is a point with measure zero.

Suppose Pi is disturbed $p(\delta Pi, 0)$ when the system is in a stationary state (all the derivatives are zeros then the initial sign vector is $(\delta Pi, \delta Po, \delta Q, \delta Xs) = (+,0,0,0)$. We will use this state-state vector notation when needed instead of an awkward notation of $p(\delta Pi,0), z(\delta Po,0), z(\delta Q,0), z(\delta Xs,0)$.

By the causal deduction, $p(\delta Q, N1)$ is first obtained(first step). Including this new state as the fact, we can then obtain $p(\delta Po, N2)$ by the causal deduction again(2nd step). Including this state as the new results and using third time fragment $dt > 0$, we obtain $n(\delta Xs, N3)$ by the causal deduction (3rd step).

3 Logical System and Dynamical System

In the previous section, we regarded the causality built-in the dynamical model as logical implication. Then, the dynamical state change can be carried out in a similar manner to deduce the new fact from the logical formulae corresponding to the dynamical model and the time fragment $p(dt, i)$. In order to use the causal relation in the dynamical model, the dynamical model must be original one. That is, the original dynamical model must reflect causal path between two physical entities.

In this section, we consider some correspondence between the important concepts in dynamical systems and those in logical systems.

Theorem 3.1(observability and deducibility)

The dynamical system is qualitative observable from a observer y iff the non-zero of the observer y can be deduced in the mapped logical system when the fact that some variables (corresponding to the dynamical system)are non-zero is given.

This result can be used to save some deduction processes when some variables are known to be observable or not. Further, this result can also used to investigate the qualitative stability which can be known by the observability of the system [Ishida 1989].

Definition 3.2 (completeness and soundness)

The mapped logical system is called complete (sound) if all the state which can (not) be attained by the corresponding dynamical system in the finite time can (not) be deduced in the finite number of steps.

Conjecture 3.3

The mapped logical system is always complete but not always sound.

This fact is often stated in qualitative reasoning, but not formally proved yet. Most formal discussion may be found in [Kuipers 1985,Kuipers 1986], stating that

> Each actual behavior of the system is necessarily among those produced by the simulation.

But,

> There are behaviors predicted by qualitative simulation which do not correspond to the behaviorof any system sarisfying the qualitative structure description.

We will see the example showing the lack of soundness of the mapped logical system in the next section. The lack of soundness is due to the following fact.

Proposition 3.4

Two equivalent dynamical systems may be mapped to the different logical systems.

That is, two dynamical systems which can be transformed to each other, may be mapped to the different logical systems. In fact, a dynamical system is usually mapped to a part of the exact logical system. Therefore, in order to make the mapped logical system close to the dynamical model, we must map from the multiple dynamical models which are equivalent as a dynamica model, and combine these mapped logical systems. We have not yet known what kinds of equivalent dynamica models suffice to make the mapped logical system exact.

4 Reasoning about State by Deduction

The causal deduction stated in the previous section cannot say anything as to changes when some time interva passed. That is, when many variables approaching to zero, which one reaches zero first. In order to determine this, meta-rules which are implicit in dynamical models must be explicitly introduced. The following axioms reflect the fact that the state-space of the dynamical models are continuous. The lack of continuous and dense space in the logical system is the fundamental points which discriminate logical systems from dynamical systems.

T-(1)

$$\forall x \forall i (p(x,i) \wedge n(dx/dt, i) \rightarrow \exists j ((j > i) \wedge z(x,j)))$$

T-(2)

$$\forall x \forall i (n(x,i) \wedge p(dx/dt, i) \rightarrow \exists j ((j > i) \wedge p(x,j)))$$

These axioms T-(1),(2) comes from value continuity rule stated in [de Kleer and Bobrow 1984]. This axiom T does not correctly reflect the world of dynamica model. Even if $x > 0$ and $dx/dt < 0$, x does not necessarily become zero in the finite or infinite time.

M-(1)

$$\forall x \forall j1 \forall j2 (p(x, j1) \land n(x, j2) \land (j2 > j1)) \rightarrow$$

$$\exists j3 (z(x, j3) \land (j3 > j1) \land (j2 > j3)))$$

M-(2)

$$\forall x \forall j1 \forall j2 (n(x, j1) \land p(x, j2) \land (j2 > j1)) \rightarrow$$

$$\exists j3 (z(x, j3) \land (j3 > j1) \land (j2 > j3)))$$

These axioms M-(1),(2) corresponds to the well-known *intermediate value theorem*, which reflects the continuity of the function x. Axioms T and M states the continuity of the state-space and that of the function from time to state-space. Other than axioms U,I,M,T, we need the following assumptions. That is, the state remains to be the same as the nearest past state unless otherwise deduced. We call this *no change assumption*. We could not formalize this assumption by a logical formula of our language so far. This seems to be a common problem to any formalization for reasoning about such dynamic concepts as state change, actions, and event. The situation calculus [McCarthy and Hayes 1969], for example, uses the *Frame Axioms*[1] to avoid this problem.

Example 4.1.

Let us consider the mass-spring system with friction [de Kleer and Bobrow 1984] (Fig. 2) whose model is of the form:

(4-1) $dx/dt = v$

(4-2) $dv/dt = -kx - fv$ where k and f are positive constants.

(4-2) is the original form containing the built-in causality whereas (4-1) is the definition of v.

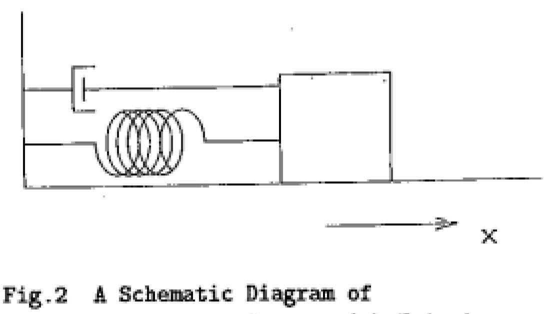

Fig.2 A Schematic Diagram of Mass-Spring System with Friction

As for the initial sign patterns of $(x, v, dv/dt)$, we consider only three cases; $(+,-,-), (+,+,-), (+,-,+)$. Let G_{dm} denote the set of logical formulae corresponding to the dynamical model, and G_{ch} those corresponding to the axioms U,I,T,M. The sign pattern $(+,+,+)$ and its opposite pattern $(-,-,-)$ are are inconsistent, since $(p(x,0) \land p(v,0)) \cup G_{dm} \rightarrow n(dv/dt, 0)$. This result $n(dv/dt, 0)$ is inconsistent with the initial pattern $p(dv/dt, 0)$ under the uniqueness axiom U.

We do not consider the initial sign pattern which contains zero for any variables, since the pattern will change immediately to the sign pattern with only non zero patterns by axiom I. Thus these three patterns cover all the possible sign combinations.

We only show the deduction for the simulation of the case 1 when $p(x,0)$, $n(v,0)$, $n(dv/dt,0)$ are given as the initial pattern. Other cases can be deduced in a similar manner to this case 1 from the initial sign pattern, the set of logical formulae G_{dm} and G_{ch}. By the axiom T,

$$p(x,0) \land n(v,0) \rightarrow \exists N1((N1 > 0) \land z(x, N1)$$

By the *no change assumption*, other variables are assumed to remain the nearest past signs; that is

$$n(v, N1), n(dv/dt, N1).$$

However,

$$(n(v, N1) \land z(x, N1)) \cup G_{dm} \rightarrow p(x, N1).$$

Thus, we have

$$z(x, N1),\ n(v, N1),\ p(dv/dt, N1).$$

Then by the axiom M,

$$(N1 > 0) \land n(dv/dt, 0) \land p(dv/dt, N1) \rightarrow \exists N2((N2 > 0) \land (N1 > N2) \land (z(dv/dt, N2))$$

By the *no change assumption*, other variables at time N2 are assumed to remain the nearest past signs; that is $p(x, N2)$, $n(v, N2)$. Since $n(v, N2) \land z(dv/dt, N2) \cup G_{dm} \rightarrow p(d^2v/dt^2, N2)$

and by the axiom I,

$$p(d^2v/dt^2, N2) \land z(dv/dt, N2) \land (N1 > N2) \rightarrow \exists N3((N1 > N3) \land (N3 > N2) \land p(dv/dt, N3)).$$

Again by the *no change assumption*,

$$p(x, N3), n(v, N3).$$

By applying the axiom I to the state at N1,

$$n(v, N1) \land z(x, N1) \rightarrow \exists N4((N4 > N1) \land p(x, N4)).$$

$n(v, N4)$ and $p(dv/dt, N4)$ are obtained by the *no change assumption*. By applying the axiom T to the

[1] Frame axioms are collection of statements that do not change when an action is performed.

state N4,

$n(v, N4) \wedge p(dv/dt, N4) \rightarrow \exists N5((N5 > N4) \wedge (z(v, N5))$.

Again signs of other variables at N5 remain to be the same as those at N4. By applying the axiom I to the state N5, we have

$z(v, N5) \wedge p(dv/dt, N5) \rightarrow \exists N6((N6 > N5) \wedge p(v, N6))$

In summary we have deduced, the set of the state at different time $p(x, N2)$, $n(v, N2)$, $z(dv/dt, N2)$, $p(x, N3)$, $n(v, N3)$, $p(dv/dt, N3)$, $z(x, N1)$, $n(v, N1)$, $p(dv/dt, N1)$,
$n(x, N4)$, $n(v, N4)$, $p(dv/dt, N4)$, $n(x, N5)$, $z(v, N5)$, $p(dv/dt, N5)$, $n(x, N6)$, $n(v, N6)$, $p(dv/dt, N6)$ and the order of time ($0 < N2 < N3 < N1 < N4 < N5 < N6$). Tables 1 show the state transitions starting the initial patterns case 1, case2 and case3.

Tables 1 State Transition by Logical Deduction

case 1

t	x	dx/dt	d^2x/dt^2
0	+	-	-
1	+	-	0
2	+	-	+
3	0	-	+
4	-	-	+
5	-	0	+
6	-	+	+

At step 6, the opposite pattern of the initial pattern comes.

case 2

t	x	dx/dt	d^2x/dt^2
0	+	+	-
1	+	0	-
2	+	-	-

At step 2, the same pattern as the initial pattern of case 1 comes.

case 3

t	x	dx/dt	d^2x/dt^2
0	+	-	+
1	0	-	+
2	-	-	+

At step 2, the opposite pattern of the initial pattern of case 2 comes.

In the logical system mapper from the dynamical model (4-1) and (4-2), it is impossible to deduce the state which corresponds to the convergence to the point $(x, dx/dt, d^2x/dt^2) = (0, 0, 0)$ which is attained when infinite time passed in the dynamical model. In fact, we only have periodic states as shown in Tables 1. However, the infinite sequences of deduction similar to this convergence can be found. When the initial sign pattern is $(x, dx/dt, d^2x/dt^2, ...) = (+, -, +, ...)$, apply the axiom T to $n(dx/dt, 0)$ then we have

$\exists N1((N1 > 0) \wedge z(dx/dt, N1))$.

Then applying the axiom M to this result, we will have

$\exists N2((N1 > N2) \wedge z(d^2x/dt^2, N2))$.

This application of the axiom M progressively to any higer order time derivative of x. That is we have

$\exists Ni + 1((Ni > Ni + 1) \wedge z(d^{i+1}x/dt^{i+1}, Ni + 1))$.

This is an interesting corresponding between the dynamical model and the mapped logical systems. It may suggest to introduce some operations in the logical system (other than deduction) which corresponds to the operation $\lim_{t\to\infty} x(t)$.

We will show this convergence can be deduced even in the finite step using the logical implications mapped from a different (but equivalent) dynamical model. The dynamical model (4-1), (4-2) is equivalent to the dynamical model:

(4-3) $E = x^2 + 1/k * (dx/dt)^2$
(4-4) $dE/dt = -f(dx/dt)^2$

This states that E and hence x will eventually become zero as long as $f > 0$. Table 2 shows the state transition of the mapped logical system. This convergence of the dynamical system is attained in the infinite time, and hence need not be deduced in the mapped logical system. Since the current logical system does not have the concepts of convergence and infinite step, these concepts are out of scope of the mapped logical systems.

The results show that the logical system mapped from the dynamical model (4-3)(4-4) is quite differet from that mapped from the dynamical model (4-1)(4-2), althogh these dynamical models are equivalent. Therefore, this example shows the correctness of Proposition 3.3. This point is also fundamental difference between dynamica systems and logical systems.

Table 2 State Transition of Mass-Spring System(Energy Model)

t	x	dx/dt	E	dE/dt
0	*	*	+	-
1	0	0	0	0

* denotes any sign +, - 0.

5 Discussions

We first discuss the *temporal logic* with the temporal operators F,P [Rescher and Urquhart 1971], where FA (PA) means A will(was) be true at some(past) future time. With the axiom schemata, the feature of these temporal operators, and even the features of time (e.g. whether it is transitive, dense, continuity) can be characterized. However, since the logic does not tell anything about the feature of the state-space and the relation between the state-space and time, it is not possible to infer the change in the state-space. In fact, the axioms I, T, M given at section ? characterize the feature of the state-space. An alternative to our approach is to define the space operators similar to the time operators. One way of defining space operator follows:

F_x, P_x where $F_xA(P_xA)$ means that A is true at some point where x is larger(smaller) than the current value. With this definition, the previous time operator can be written as F_t, P_t.

With these space operators, the axioms I,T,M may be written as:

I-(1) z(x) $\rightarrow G_x(p(x))$
I-(2) z(x) $\rightarrow H_x(n(x))$
T-(1) p(x) $\rightarrow P_x(z(x))$
T-(2) n(x) $\rightarrow F_x(z(x))$
M-(1) p(x) $\wedge F_x(n(x)) \rightarrow F_x(F_x(z(x)))$
M-(2) n(x) $\wedge F_x(p(x)) \rightarrow F_x(F_x(z(x)))$

Since these axiom schemata I,T,M characterize the feature of only state-space itself, we need the following axioms TS which characterize the monotonic relation between time and state-space.

TS-(1) p(dx/dt) $\rightarrow ((FA \leftrightarrow F_xA) \wedge (PA \leftrightarrow P_xA))$
TS-(2) n(dx/dt) $\rightarrow ((PA \leftrightarrow F_xA) \wedge (FA \leftrightarrow P_xA))$

where $G_xA \leftrightarrow \sim F_x \sim A, H_xA \leftrightarrow \sim P_x \sim A$.

Here, the time operators are used instead of the time index for the sign predicates p,n,z. The good point of this space operator approach is that it can be discussed as a natural extension of temporal logic with temporal operators. However, its critical point is that although these space/time operators can tell the temporal precedence of the event but it cannot describe that the different event A, B occurred at the same time. In the approach taken in section 4, it is described by putting the same time tags.

When compared with the qualitative reasoning [de Kleer and Bobrow 1984], our way of qualitative reasoning is different from theirs in the following two points: (1) In reasoning; we defined another causality which refers to time strictly. Causal reasoning is carried out by mapping causality in dynamical models to the deduction under the condition of $dt > 0$. Time independent relations are mapped to only deductions. Then causa reasoning is done by requiring the facts $dt > 0$ in every step. This logical reasoning can be implemented on the the logical reasoning system such as prolog by providing axioms so far proposed and the mapped dynamical models. (2) In modeling; since we use the causality built in the dynamical model, we skip qualitative modeling process. That is, we use the dynamical model as qualitative model. However, the dynamical models must be carefully selected to insure the causal path in the dynamica models can be reflected on the mapped logical systems.

6 Conclusion

We discussed a mapping from dynamical systems to logical systems to see the correspondence of the fundamental concepts in these two domains, to implement the causal reasoning system on a logical deduction system. To clearly separate the physical causality from the usua deduction, we defined causality in physical system by making time explicit.

Many fundamental problems remains such as; whether or not the complete and sound logical system for a dynamical system exists ? If yes, how the complete and sound logical system can be attained?

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