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Logical Structure of Analogy

PRELIMINARY REPORT

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Abstract: This paper treats a general type of analogical reasoning which is described as follows: when two objects, B (the*base*) and T (the *target*), share a property S (the *similarity*), it is conjectured that T satisfies another property P (the *projected property*) which B satisfies as well.

Through a formal analysis of this type of analogy, a logical relation is explored which is necessarily satisfied by the tuple, T, B, S, P, under an axiom, $\mathcal{A}$. Unlike previous studies on analogy, this work does not give any particular assumption a priori to the tuple.

By the analysis, it is shown to be reasonable that analogical reasoning is possible only if a certain form of rule, called the *analogy prime rule*, is a deductive theorem of a given theory, and that, from the rule, together with two particular conjectures, an analogical conclusion is derived. Also, a candidate is shown for a non-deductive inference system which can yield both conjectures.

1 Introduction

When we explain a process of reasoning by analogy, we may say, "An object T is similar to another object B in that T shares a property S with B and B satisfies another property P. Therefore, T also satisfies P". We may express this more formally using the following schema.

$$\frac{\begin{array}{c} S(B) \wedge P(B) \\ S(T) \end{array}}{P(T)}$$

Here, T will be called the *target*, B the *base*, S the *similarity* between T and B, and P the *projected property*.

The above description of the process of analogy is, however, insufficient. Researchers studying analogy have come to recognize the necessity of revealing some implicit condition which influences the process but does not appear in the above schema. The importance of this has already been discussed enough in [3]. The implicit condition to be satisfied by appropriate analogical factors, T, B, S, and P, can, formally, be characterized only by a given theory (axiom), written as $\mathcal{A}$. The objective of this paper is to explore the particular relation of analogy which T, B, S, P and $\mathcal{A}$ necessarily satisfy.

In the study of analogy, the following have been central problems:

1) what object should be selected as a base w.r.t a target,

2) which property is significant in analogy among properties shared by two objects, and

3) what property is to be projected w.r.t. a certain similarity.

Many significant works have been vigorously conducted on these problems, though they were only partially successful in answering these questions, that is, by giving intuitive and strong assumptions a priori. In many works, a base case was assumed to be given w.r.t. a target case [4, 11, 10]. In almost all works, the important similarity (or similarity measure) was defined a priori independently of what property was projected [20, 6, 10, 7, 5]. In logical works [8, 5], especially in [3], nice logical relations among the analogical factors could be seen, though they, like others, were given without sufficient examinations which would show *why* and *how* their relations were *necessary*.

Unlike previous studies on analogy, this work does not give any particular assumption a priori to the analogical factors. Clarifying the relation between the factors, T, B, S, P and $\mathcal{A}$, will be enough to answer the above three problems once and for all. The relation shown by this paper is a general solution for them and might show how useful a formal treatment is in analyzing analogical behavior.

First, through a logical analysis of analogy, it is shown to be reasonable that, when an analogical inference is done under a theory $\mathcal{A}$, a particular form of rule must be a logical conclusion (a theorem) of $\mathcal{A}$ and that analogical inference is accomplished by two particular types of (generally non-deductive) conjectures. Then, a non-deductive inference is proposed, which is shown to be an adequate candidate to yield the conclusions of both these conjectures.

2 A Logical Analysis

2.1 Preparations

In this paper, we use standard formal logic and notations, while defining the following. An n-ary *predicate* U is generally expressed by λxQ, where x is a tuple of n object variables, Q is a formula in which no object variables except variables in x occur free. If t is a tuple of n terms, $U(t)$ stands for the result of replacing each occurrence of (elements of) x in Q with (each corresponding element of) t simultaneously. For any formulas A and F, when $A \vdash F$ and $\not\vdash F$ (that is, F is not *valid*), we say F is a *genuine* theorem of A and express it simply as $A \mid\!\!-F$.

We will use a closed formula of first order logic $\mathcal{A}$ for a *theory*, (generally n) terms T for a *target* and (generally n) terms B for a *base*. A property is expressed by a predicate, for instance, a *similarity* and a *projected property* are expressed by predicates, S and P respectively.

2.2 Approach To A Seed of Analogy

We can understand analogical reasoning as follows:

(1) Example-based Information: "An object, x' (corresponding to a base), satisfies both properties S and P $(\exists x'.(S(x') \wedge P(x')))$."

(2) Similarity-based Information: "Another object, x (corresponding to a target), satisfies a shared property S with x' $(S(x))$."

(3) Analogical Conclusion: "The object x would satisfy the other property P $(P(x))$."

Then,

"Analogical reasoning is to reason (3) from $\mathcal{A}$ together with (1)+(2)." (A)

Let this understanding be our starting point of analysis.

As analogy is not, generally, deductive, this starting point may, unfortunately, be expressed only as follows. In the notation of proof theory,

$$\mathcal{A}, \exists x'.(S(x') \wedge P(x')), S(x) \not\vdash P(x). \tag{1}$$

As analogy, however, infers $P(x)$ from the premises, it implies that some knowledge is assumed in the premise part of (1). Let the assumed knowledge be $F(x)$, providing that it depends on the x in general. That is,

$$\mathcal{A}, \exists x'.(S(x') \wedge P(x')), S(x), F(x) \vdash P(x). \tag{2}$$

Thus, the essential information newly obtained by analogy is $F(x)$ in the above rather than the explicit projected property P . Making $J(x)$ stand for the conjunction of the example-based information and $F(x)$, the above meta-sentence is transformed equivalently to

$$\mathcal{A} \vdash \forall x.(J(x) \wedge S(x) \supset P(x)), \tag{3}$$

because $\mathcal{A}$ is closed. This implies that a rule must be a theorem of $\mathcal{A}$ and that the rule concludes any object which satisfies $J(x)$ to satisfy P when it satisfies S. Once J is satisfied, (by reason of $(S(x) \supset P(x))$,) the analogical conclusion ("an object satisfies P") can be deduced from the similarity-based information ("the object satisfies S). For this reason, this rule will be called the *analogy prime rule* (it will be specified in more detail later), J will be called the *analogy justification*.

Moreover, it is improbable that the analogy prime rule is a valid formula, because, if so, any pair of predicates can be an analogical pair of a similarity and a projected property independently of $\mathcal{A}$. Thus, the analogical prime rule must be a genuine theorem of $\mathcal{A}$,

$$\mathcal{A} \mid\!\!-\forall x.(J(x) \wedge S(x) \supset P(x)). \tag{4}$$

Consequently, an object T which satisfies S is concluded to satisfy P from an analogy prime rule by analogical reasoning that assumes that T satisfies the analogy justification ($J(T)$). That is, our starting point (A) can be specified from two aspects.

"An analogical conclusion can be obtained from an analogy prime rule together with example-based information and similarity-based information." (B)

"A non-deductive jump by analogy, if it occurs, is to assume that the analogy justification of the prime rule is satisfied." (C)

In the following part of this paper, the analogy justification and non-deductivity will be further explored. Before beginning an abstract discussion, it may be useful to see concrete examples of analogical reasoning. The next section introduces "target" examples of analogical reasoning to be clarified here.

2.3 Examples

Example1: Determination Rule[3]. "Bob's car (C_{Bob}) and Sue's car (C_{Sue}) share the property of being 1982 Mustangs (*Mustang*). We infer that Bob's car is worth about \$3500 just because Sue's car is worth about \$3500. (We could not, however, infer that Bob's car is painted red just because Sue's car is painted red.)"
Example-based Information:

$$Model(C_{Sue}, Mustang) \wedge Value(C_{Sue}, \$3500), \tag{5}$$

Similarity-based Information:

$$Model(C_{Bob}, Mustang), \quad (6)$$

Example2: Brutus and Tacitus [1]. " Brutus feels pain when he is cut or burnt. Also, Tacitus feels pain when he is cut. Therefore, if Tacitus is burnt, he will feel pain."

Example-based Information:

$$(Suffer(Brutus, Cut) \supset FeelPain(Brutus)) \quad (7)$$
$$\wedge(Suffer(Brutus, Burn) \supset FeelPain(Brutus)) \quad (8)$$

Similarity-based Information:

$$Suffer(Tacitus, Cut) \supset FeelPain(Tacitus) \quad (9)$$

Example3: Negligent Student[1]. " When I discovered that one of the newcomers ($Student_T$) to our laboratory was a member of an orchestra club ($Orch$), remembering that another student ($Student_B$) was a member of the same club and he was often negligent of study ($Study$), I guessed that the newcomer would be negligent of study, too."

Example-based Information:

$$Member\_of(Student_B, Orch)$$
$$\wedge Negligent\_of(Student_B, Study) \quad (10)$$

Similarity-based Information:

$$Member\_of(Student_T, Orch) \quad (11)$$

2.4 Logical Analysis: a rule as a seed of analogy

In treating analogy in a formal system, as the information of a base object being S and P is projected into a target object, it is desirable to treat such properties as *objects* so that we can avoid the use of second order language. As an example, the fact that Bob's car is a Mustang is represented by "$Model(C_{Bob}, Mustang)$" rather than simply as "$Mustang(C_{Bob})$". In the remaining part, we rewrite $S(x)$ to $\Sigma(x, S)$ and $P(x)$ to $\Pi(x, P)$. Σ will be called a *similar attribute*, Π will be a *projected attribute*, S as an object will be a *similar attribute value*, and P as an object will be a *projected attribute value*. Then, (4) is rewritten

$$\mathcal{A} \vdash \forall x, s, p.(J(x, s, p) \wedge \Sigma(x, s) \supset \Pi(x, p)), \quad (12)$$

considering the most general case that the analogy justification J depends on all of these factors.

Again, when 3-tuple < object: X, similar attribute value: S, projected attribute value: P > satisfies the analogy justification J, object X is conjectured to satisfy the projected property $\lambda x.\Pi(x, P)$ (analogical conclusion) just because X has the similarity $\lambda x.\Sigma(x, S)$. That is, $J(x, s, p)$ can be considered a condition, where x could be concluded to be p from x being s by analogical reasoning.

[1]The author thanks Satoshi Sato (Hokuriku Univ.) for showing this challenging example.

Now, recalling that an analogical conclusion is obtained from the analogy prime rule with example-based information and similarity-based information, consider what information can be added by the information in relation to the analogy prime rule.

1) **Example-based Information:** This shows that there exists an object as a base which satisfies a similarity and a projected property ($\exists x'.(\Sigma(x', S) \wedge \Pi(x', P))$). It seems to be adequate that the base, B, satisfying $\Sigma(x', S)$ can also be derived to satisfy $\Pi(x', P)$ from the prime rule, because B can be considered a target which has similarity S. That is, 3-tuple $< B, S, P >$ satisfies the analogy justification. Consequently, from arbitrariness in selection of an object as a base in this information, what is obtained from this information is $\exists x'.\ J(x', S, P)$.

2) **Similarity-based Information:** This shows that an object as a target, T, satisfies the same property S in the above. Just by this fact, an analogical conclusion is obtained, by assuming that the object satisfies J by some conjecture. That is, there exists some attribute value p' and 3-tuple $< T, S, p' >$ satisfies J ($\exists p'.\ J(T, S, p')$).

3) **Analogical Conclusion:** With the above two pieces of information, an analogical conclusion, "T satisfies $\Pi(x, P)$", is obtained from the analogy prime rule. Therefore, such 3-tuple $< T, S, P >$ satisfies J ($J(T, S, P)$).

In the above discussion, T, S, and P are arbitrary. Therefore, the following relation about the analogy justification turns out to be true:

$$\forall x, s, p.(\ \exists x'.J(x', s, p) \wedge \exists p'.J(x, s, p') \supset J(x, s, p)\ ). \quad (13)$$

(13) is able to represent it equivalently as follows:

$$J(x, s, p) \ = \ J_{att}(s, p) \wedge J_{obj}(x, s), \quad (14)$$

where both J_{att} and J_{obj} are predicates, that is, each of them has no free variables other than its arguments.

The point shown by this result is that any analogy justification can be represented by a conjunction in which variable x and variable p occur separately in different conjuncts.

By (12) and (14), the analogical prime rule can be defined as follows.

Definition 1 Analogy Prime Rule
A rule is called an analogy prime rule w.r.t. $< \Sigma(x, s); \Pi(x, p) >$, *if it has the following form:*

$$\forall x, s, p. (J_{att}(s,p) \wedge J_{obj}(x,s) \wedge \Sigma(x,s) \supset \Pi(x,p)), \quad (15)$$

where J_{att}, J_{obj}, Σ and Π are predicates. (That is, each of $J_{att}(s,p)$, $J_{obj}(x,s)$, $\Sigma(x,s)$ and $\Pi(x,p)$ is a formula in which no variable other than its arguments occurs free.) □

In (15), $J_{att}(s,p)$ will be called the *attribute justification* and $J_{obj}(x,s)$ will be called the *object justification.*

Also, by the above discussion, the following two conjectures can be considered as causes which make analogy non-deductive.

- **Example-based Conjecture (EC):** An object shows a existing concrete combination of a similarity and a projected property. This specializes the prime rule and allows it to be applicable to a similar object. Assuming some generally non-deductive inference system under $\mathcal{A}$, "$\mid\!\sim^{\mathcal{A}}$" (we will propose such a system later),

$$\exists x. (\Sigma(x,S) \wedge \Pi(x,P)) \mid\!\sim^{\mathcal{A}} J_{att}(S,P). \quad (16)$$

- **Similarity-based Conjecture (SC):** Just because an object satisfies S, application of the specialized prime rule to the object is allowed.

$$\Sigma(x,S) \mid\!\sim^{\mathcal{A}} J_{obj}(x,S). \quad (17)$$

In case that the attribution justification ($J_{att}(s,p)$) is a valid formula, example-based information becomes unnecessary in yielding analogical conclusion. Thus, it could, in general, be essential in analogical reasoning to guess $J_{att}(s,p)$ which is not a valid formula. The object justification ($J_{obj}(x,s)$) is, still, important in another sense, because it can be considered to express a *really* significant similarity. It is not an unusual case when a really significant similarity is not observable. Consider a case of Example 2. Having a nervous system will be a sufficient condition for an object to feel pain, thus, whether an object has a nervous system is a significant factor in making a conjecture on feeling pain. In this case, however, we could, without dissection, not obtain a direct evidence which shows that Tacitus and Btutus have nervous systems, while we obtain only a *circumstantial* evidence that the both feel pain when they are cut. Thus, the similarity-based conjecture is to guess such a really significant but implicit similarity, the object justification ($J_{obj}(x,s)$), from an observed similarity $\Sigma(x,s)$.

To summarize, a logical analysis of analogy could draw conclusions as follows.

Analogical reasoning is possible only if a certain *analogical prime rule* is a *genuine* theorem of a given theory and the process of analogical reasoning can be divided into the following 3 steps: 1) the attribute justification part of the rule is satisfied by EC from example-based information, 2) the object justification part of the rule is satisfied by SC from similarity-based information, and, 3) from similarity-based information and the analogy prime rule specialized by the two preceding steps, an analogical conclusion is obtained by deduction.

A question remains unclear, that is, what inference is EC and what SC? Though we cannot identify the mechanism underlying each of the conjectures, we can propose a (generally) non-deductive inference system as their candidates. The next section shows this.

3 Non-deductive Inference for Analogy

This section explores a type of generally non-deductive inference by which a conjecture G is obtained from a given theory $\mathcal{A}$ with additional information K.

Generally speaking, what properties should be satisfied by a, generally, non-deductive inference? It might be desirable that a non-deductive inference satisfies at least the following conditions. First, it should subsume deduction, that is, any deductive theorem is one of its theorems, because any deductive conclusion would be desirable. Secondly, any conclusion obtained by it must be able to be used deductively, that is, from such a conclusion, it should be possible to yield more conclusions using, at least, deduction. And, thirdly, any conclusion obtained must be consistent with given information. We define a class of inference systems which satisfy the above three conditions.

Definition 2 *An inference system under a theory $\mathcal{A}$ (written $\mid\!\sim^{\mathcal{A}}$) is deductively expansible if the following conditions are satisfied. For any set of sentences $\mathcal{A}$ and K and any sentences G and H,*

i) *Subsuming deduction:*

if $\mathcal{A}, K \vdash G$ *then* $K \mid\!\sim^{\mathcal{A}} G$.

ii) *Deductive usefulness:*

if $K \mid\!\sim^{\mathcal{A}} G$ *and* $\mathcal{A}, K, G \vdash H$, *then* $K \mid\!\sim^{\mathcal{A}} H$.

iii) *Consistency:*

if $K \mid\!\sim^{\mathcal{A}} G$ *and* $\mathcal{A} \cup K$ *is consistent, then* $\mathcal{A} \cup K \cup \{G\}$ *is consistent.*

The following inference system is an example of a deductively expansible system.

Definition 3 *G is a conjecture from $\mathcal{A}$ based on K by (atomic) circumstantial reasoning (written $K \mid\!\sim^{\mathcal{A}}_{*} G$)* [2]*, iff*

i) $\mathcal{A}, K \vdash G$, *or*

ii) $\mathcal{A}, E \vdash G$
if there exists a minimal set of atomic formulas[3] *E s.t.* $\mathcal{A}, E \vdash K$, *and* $\mathcal{A} \cup E$ *is consistent if* $\mathcal{A} \cup K$ *is consistent*[4].

Proposition 1
If $K \mid\!\sim^{\mathcal{A}} G$ *and* $K, G \mid\!\sim^{\mathcal{A}}_{*} H$, *then* $K \mid\!\sim^{\mathcal{A}} H$.

Corollary 1 *If* $K \mid\!\sim^{\mathcal{A}}_{*} G$, *then* $K \mid\!\sim^{\mathcal{A}} G$.

Corollary 1 shows that circumstantial reasoning is deductively expansible, and proposition 1 (together with the corollary) shows that inference done by multiple applications of circumstantial reasoning is also deductively expansible.

Circumstantial reasoning ($K \mid\!\sim^{\mathcal{A}}_{*} G$) implies a very general and useful inference class in that so many types of inference used in AI can be considered as circumstantial reasoning. Deduction and abduction, for example, are obviously circumstantial reasoning. Moreover, if we loosen the condition "atomic formulas" to "clauses", inductive learning from examples is the case where $\mathcal{A}$ is empty in general, K is "examples" and G is inductive knowledge obtained by "learning"[5] [6]

Now, we assume that both EC and SC are circumstantial reasoning, but based on different information. Then, we can see analogical reasoning in more detail.

Let an analogy prime rule w.r.t. $< \Sigma(x,s); \Pi(x,p) >$ be a theorem of $\mathcal{A}$. Then, when example-based information, $\Sigma(B,S) \wedge \Pi(B,P)$, is introduced, by circumstantial reasoning from the prime rule, some justifications are satisfied, that is,

$$\Sigma(B,S) \wedge \Pi(B,P) \mid\!\sim^{\mathcal{A}}_{*} J_{att}(S,P) \wedge J_{obj}(B,S), \quad (18)$$

which concludes a specialized prime rule,

$$\forall x.(J_{obj}(x,S) \wedge \Sigma(x,S) \supset \Pi(x,P)). \quad (19)$$

Even if similarity-based information $\Sigma(T,S)$ is introduced, to obtain analogical conclusion $\Pi(T,P)$ by circumstantial reasoning, some information apart from the prime rule turns out to be needed in $\mathcal{A}$. And, both EC and SC are generally needed to accomplish analogical reasoning, which implies that multiple application of circumstantial reasoning is necessary. Even in such a case, circumstantial reasoning remains worthwhile (Proposition 1).

4 Classification of Analogy and Examples

Each EC and SC has two cases: a deductive one and a non-deductive one. According to this measure, analogical inference can be divided into 4 types. A typical example is shown in each class and explored.

4.1 deductive EC + deductive SC

Typical reasoning of this type was proposed by T.Davies and S.Russell [3]. They insisted that, to justify an analogical conclusion and to use information of the base case, a type of rule, called a *determination rule*, should be a theorem of a given theory. The rule can be written as follows:

$$\forall s,p.(\ \exists x'.(\Sigma(x',s) \wedge \Pi(x',p)) \supset \forall x.(\Sigma(x,s) \supset \Pi(x,p))\ ) \quad (20)$$

Example 1 (continued). In this example, the following determination rule is assumed to hold under $\mathcal{A}$.

$$\forall s,p.(\ \exists x'.(Model(x',s) \wedge Value(x',p)) \supset \forall x.(Model(x,s) \supset Value(x,p))\ ) \quad (21)$$

This rule is an analogy prime rule, because

$J_{obj}(x,s) = \Sigma(x,s) = Model(x,s)$,
$J_{att}(s,p) = (\exists x.\ Model(x,s) \wedge Value(x,p))$,
$\Pi(x,p) = Value(x,p)$.

Moreover,

EC: $$Model(C_{Sue}, Mustang) \wedge Value(C_{Sue}, \$3500) \vdash J_{att}(Mustang, \$3500), \quad (22)$$

SC: $$Model(C_{Bob}, Mustang) \vdash J_{obj}(C_{Bob}, Mustang). \quad (23)$$

This illustrates that reasoning based on determination rules belongs to the "deductive EC + deductive SC" type and that it can also be done by circumstantial reasoning.

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[2] Circumstantial reasoning is essentially equivalent to "abduction" + deduction [13, 15]. However, "abduction" has many definitions and various usages in different contexts, so we like to introduce a new term for the type of inference in Definition 3 to avoid confusion.

[3] *Atoms*, that is, formulas which contain only one predicate symbol.

[4] If there exists such a minimal set of atomic formulas E, the case ii) involves the case i) apparently. Thus, the case i) can often be neglected in a usual application, for instance, if K is a *universal* formula which has the form $\forall x.F(x)$, where F is quantifier-free. Note that a clause is universal.

[5] In this case, $G = E$ in Definition 3, which implies that G is a minimal set to explain "example" K. Indeed, such minimality is very common in this field.

[6] Such a unified aspect of various reasoning in AI was pointed out by Koich Furukawa (ICOT) in a private discussion and a similar and more intuitive view can be seen in [5].

4.2 deductive EC + non-deductive SC

This type of analogical reasoning was explored by the author [1]. It was concluded that, once we assumed the following two premises for analogical reasoning, it seemed to be *an inevitable conclusion* that analogical reasoning which infers $P(T)$ from $S(T)$, $S(B)$, and $P(B)$ satisfies *the illustrative criterion*. And if an inference system satisfies the criterion, the system is called an *illustrative analogy*.

Premise 1: "Analogy is done by projecting properties (satisfied by a base) from the base onto a target."

Premise 2: "The target is not a special object."

Premise 2 is also assumed in this paper, it is translated into an arbitrary selection of a target object. Premise 1 was translated as follows: $J(B)$, (where J is the justification in (4) and B stands for a base object) must be a theorem of $\mathcal{A}$, because it is essential in analogical reasoning to project $J(B)$ onto a target object T. That is, the non-deductive part in this reasoning is just SC which conjectures the property of the target object, and EC must be deductive.

Example 2 (continued). By illustrative analogy, a target is conjectured to satisfy properties used in an explanation of why a base satisfies a similarity. In this example, to explain the phenomena of the base case, "Brutus feels pain when he is cut or burnt", the following sentences must be in $\mathcal{A}$.

$$\forall x, i. (\ Nervous\_Sys(x) \land Destructive(i) \land Suffer(x, i) \supset FeelPain(x)\ ), \quad (24)$$

$$\land Nervous\_Sys(Brutus) \quad (25)$$

$$\land Destructive(Cut) \land Destructive(Burn) \quad (26)$$

From (24), the following follows:

$$\forall x, s, p. (\ Nervous\_Sys(x) \land Destructive(s) \land Destructive(p) \land (Suffer(x, s) \supset FeelPain(x)) \supset (Suffer(x, p) \supset FeelPain(x))\ ). \quad (27)$$

which is an analogy prime rule, that is,

$J_{obj}(x, s) = Nervous\_Sys(x)$,
$J_{att}(s, p) = Destructive(s) \land Destructive(p)$,
$\Sigma(x, s) = Suffer(x, s) \supset FeelPain(x)$,
$\Pi(x, p) = Suffer(x, p) \supset FeelPain(x)$.

$J_{att}(Cut, Burn)$ ("Both cut and burn are destructive") is a deductive theorem of $\mathcal{A}$ and a non-deductive conjecture, $J_{obj}(Tacitus, Cut)$ ("Tacitus has a nervous system"), is obtained by circumstantial reasoning from (24) based on the similarity-based information, $Suffer(Tacitus, Cut) \supset FeelPain(Tacitus)$.

4.3 non-deductive EC + deductive SC

As far as the author knows, this type of analogy has never been discussed. Example 3 seems to show this type of analogy.

Example 3 (continued). First, let us consider what we know from example-based information in this case. From the fact that a student ($Student_B$) was a member of the same club ($Orch$) and often neglected study ($Study$), we could find that "the orchestra club keeps its members very busy ($BusyClub(Orch)$)" and that "activities of the club are obstructive to one's study ($Obstructive\_to(Orch, Study)$)". This implies that we knew some causal rule like "If it is a busy club and its activities are obstructive to something, then any member of the club neglects the thing."

$$\forall x, s, p. (\ BusyClub(s) \land Obstructive\_to(p, s) \land Member\_of(x, s) \supset Negligent\_of(x, p)\ ) \quad (28)$$

Using this rule, we found the above information.

Thus, the above rule is assumed to be a theorem of $\mathcal{A}$. $BusyClub(Orch)$ and $Obstructive\_to(Orch, Study)$ are non-deductive conjectures and it can be obtained by circumstantial reasoning based on the above rule which is just an analogy prime rule, as follows:

$J_{obj}(x, s) = \Sigma(x, s) = Member\_of(x, s)$,
$J_{att}(s, p) = BusyClub(s) \land Obstructive\_to(p, s)$,
$\Pi(x, p) = Negligent\_of(x, p)$.

4.4 non-deductive EC + non-deductive SC

As an example of this type, we can take Example 2 again. We might know neither "Brutus has a nervous system" nor "Both cut and burn are destructive", which corresponds to the case that (25) and (26) are not in $\mathcal{A}$ (nor any deductive theorem of $\mathcal{A}$) in the previous Example 2. However, by circumstantial reasoning from (24) based on example-based information ("Brutus feels pain when he is cut or burnt"), "Both cut and burn are destructive" (and "Brutus has a nervous system") can be obtained, and based on similarity-based information ("Tacitus feels pain when he is cut"), "Tacitus has a nervous system", a really significant but implicit similarity, is obtained similarly to the previous example. Consequently, the analogical conclusion ("Tacitus would feel pain when he is burnt") is derived from (27) (or (24)) together with the above conjectures.

5 Conclusion and Remarks

- Through a logical analysis of analogy, it is shown to be reasonable that analogical reasoning is possible only if a certain *analogy prime rule* is a deductive theorem of a given theory. From the rule, together with *an example-based conjecture* and *a similarity-based conjecture, the analogical conclusion* is derived. A candidate is shown for a non-deductive inference system which adequately yields both conjectures.

- Results shown here are general and do not depend on particular pragmatic languages like the *purpose* predicate [10] nor on some numeric similarity measure [20]. These results can be applied to any normal deductive data bases (DDB) which consist of logical sentences.

  Application of this analogical reasoning to DDB may be one of the most fruitful. It is, generally speaking, very difficult to build a DDB which involves perfect knowledge about an item. Analogical reasoning will increase the chance of answering queries adequately, even when its deductive operation fails to answer. In a DDB, it is very common to see *inheritance* rules and *transitivity(-like)* rules, which have the form of the analogy prime rule, for instance,

$$\texttt{Gran\_pa(x,y)} : -\texttt{Parent(x,z)},\texttt{Parent(z,y)}. \quad (29)$$

  This is an analogy prime rule w.r.t. $<\texttt{Parent(z,y)};\texttt{Gran\_pa(x,y)}>$ (z is a variable for the similar attribute value and x is a variable for the projected attribute value). Assume that a query "?-Gran_pa(x, Tom)" is given to a database $\mathcal{A}$ which involves the above rule and the following facts:

$$\texttt{Parent(Sue,Tom)}. \quad (30)$$

$$\texttt{Gran\_pa(John,Bob)}. \quad (31)$$

$$\texttt{Parent(Sue,Bob)}. \quad (32)$$

  The database cannot answer the query deductively, because it does not know who is a parent of Sue. If the database uses the proposed type of analogical reasoning, it is able to guess Gran_pa(John, Tom) from Bob's case just because Tom is similar to Bob in that their parents is the same.

  Interestingly, a method which discovers an analogy prime rule from knowledge data-base CYC is explored independently[17]. Such methods make analogical reasoning more common in DDB.

- By the side effect of this analysis, it becomes possible to compare analogy with other reasoning formally which have been studied vigorously in the area of artificial intelligence. *Analogical* reasoning **differs from** other reasoning, *abductive* and *deductive*, in that analogical reasoning actually uses example-based information (the base information). Consider the difference from, this time, abduction in the above database case. Even if the database uses (ordinal) abductive reasoning in the query, it cannot specify an adequate grandparent of Tom, the possible answer will be x s.t. Gran_pa(x, Tom), Parent(x, Sue), $(\exists$z.)(Parent(x, z), Parent(z, Tom)), or Sue assuming Parent(Sue, Sue), *etc* [2, 14, 18, 9]. The reason for this failure is that abduction tries to *explain* only the target case.

  Moreover, comparing with enumerative *induction* and *case-based reasoning* (CBR) in which the use of examples are essential similarly to analogical reasoning, analogical reasoning has a salient feature in more strongly depending on a background knowledge (a given theory). Analogy can be seen as a *single instance generalization* as Davies and Russell pointed out [3]. Take an example, Example 3. From the analogy prime rule (28) and example-based information of *an* base case ($Student_B$), some non-deductive inference (ex. circumstantial reasoning) yields a more specified analogy prime rule,

$$\forall x.(\ Member\_of(x, Orch) \supset Negligent\_of(x, Study)\ ), \quad (33)$$

  which is a generalization of the example-based information,

$$Member\_of(Student_B, Orch) \wedge Negligent\_of(Student_B, Study). \quad (34)$$

  We should note that, in the process of this single instance generalization, an analogy prime rule in a background knowledge is used as an intermediary, and it might be considered the reason why analogy seems more plausible than a simple single instance generalization such that it yields (33) just from (34).

  In the research of formal inductive inference [16, 12], a back ground knowledge does not play such an important role. So, plenty of examples are needed until a plausible conclusion is obtained. Concerning CBR [19], though it uses base cases like analogical reasoning and, in order to retrieve their base cases, it uses an *index* which corresponds to the similarity S, the index is assumed to be given in spite of using background knowledge. Intuitively speaking, these methods will be very useful when a background knowledge is rather poor or difficult to formulate, and when the background knowledge is extremely strong or able to be formulated perfectly, deduction will be most useful, on the other hand, the proposed type of analogy will be useful when rather strong and difficult to formulate.

- An implementation system for this type of analogy has been developed. Given a theory $\mathcal{A}$, a target T and a projected attribute $\Pi(x,p)$ (from a query, "?- $\Pi(T,p)$"), this system finds a base B, a similarity $\Sigma(x,S)$ and a projected property $\Pi(x,P)$ (ie. "$\Pi(T,P)$" is the answer of the query) by the process with backtracking, according to the following steps:

  1) *Find a* **separate rule** $SepR$ *s.t.* $\mathcal{A} \vdash SepR$.
  where $SepR = \Pi(x,p)$:- $G_{att}(s,p), G_{obj}(x,s)$.

  2) *Take a* **similar attribute** $\Sigma(x,s)$
  s.t. $\Sigma(x,s) \mid\!\sim^{\mathcal{A}}_{*} G_{obj}(x,s)$.

  3) *Obtain the* **similar attribute value** S
  by the side effect of a proof $\mathcal{A} \vdash \exists s.\Sigma(T,s)$.

  4) *Retrieve a* **base** B *and obtain the* **projected attribute value** P
  by the side effect of a proof
  $\mathcal{A} \vdash \exists x,p.(\Sigma(x,S) \wedge \Pi(x,p))$.

  Here, a *separate rule* (*w.r.t.* $\Pi(x,p)$) is a Horn clause in which the head is $\Pi(x,p)$, and any variable of x and any variable of p does not appear in the same conjunct in the body. This system guesses successfully for the examples shown here, though each of them is translated into a set of Horn clauses.

  Significant restrictions are needed on the time complexity of this process. Details of this system will be reported elsewhere.

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References

[1] Arima, J.: A logical analysis of relevance in analogy, in *Proc. of Workshop on Algorithmic Learning Theory (ALT'91)*, (1991).

[2] Cox P.T. and Pietrzykowski T.: Causes for events: their computation and applications, in: *Proc. of Eighth International Conference on Automated Deduction*, Lecture Notes in Computer Science **230** (Springer-Verlag, Berlin, 1986) pp. 608-621.

[3] Davies, T. and Russell, S.J.: A logical approach to reasoning by analogy, in *IJCAI-87*, pp.264-270 (1987).

[4] Evans, T,G.: A program for the solution of a class of geometric analogy intelligence test questions, in: M.Minsky (Ed.), *Semantic Information Processing* (MIT Press, Cambridge, MA, 1968).

[5] Falkenhainer, B.: A unified approach to explanation and theory formation, in: J.Shrager & P.Langley (Ed.), *Computational Models of Scientific Discovery and Theory Formation*, (Morgan Kaufmann, San Mareo, CA, 1990).

[6] Gentner, D.: Structure-mapping: Theoretical Framework for Analogy, in: *Cognitive Science*, Vol.7. No.2, pp.155-170 (1983).

[7] Greiner, R.: Learning by understanding analogy, *Artificial Intelligence*, Vol. 35, pp.81-125 (1988).

[8] Haraguchi, M. and Arikawa, S: Reasoning by Analogy as a Partial Identity between Models, in *Proc. of Analogical and Inductive Inference (ALL'86)*, Lecture Notes in Computer Science **265**, (Springer-Verlag, Berlin, 1987) pp 61-87.

[9] Inoue, K.: Linear Resolution for Consequence-Finding, in *Artificial Intelligence* (To appear).

[10] Kedar-Cabelli, S.: Purpose-directed analogy, in *the 7th Annual Conference of the Cognitive Science Society*, Hillsdale, NJ: Lawrence Erlbaum Associates, pp.150-159 (1985).

[11] Kling, R.E.: A paradigm for reasoning by analogy, *Artificial Intelligence* **2** (1971).

[12] Muggleton, S. and Buntine, W.: Machine Invention of First-Order Predicates by Inverting Resolution, In: *Proc. of 5th International Conference on Machine Learning*, pp 339-352 (1988).

[13] Peirce C.S.: *Elements of Logic*, in: C. Hartshorne and P. Weiss (eds.), *Collected Papers of Charles Sanders Peirce*, Volume 2 (Harvard University Press, Cambridge, MA, 1932).

[14] Poole D., Goebel R. and Aleliunas R.: Theorist: a logical reasoning system for defaults and diagnosis, in: N. Cercone and G. McCalla (eds.), *The Knowledge Frontier: Essays in the Representation of Knowledge* (Springer-Verlag, New York, 1987) 331–352.

[15] Pople, H.E.Jr.: On the mechanization of abductive logic, in: *Proceedings IJCAI-73*, Stanford, CA (1973) 147–152.

[16] Shapiro, E.Y.,: Inductive Inference of Theories From Facts, TR 192, Yale Univ. Computer Science Dept. (1981).

[17] Shen ,W.:Discovering Regularities from Knowledge Bases, *Proc. of Knowledge Discovery in Databases Workshop 1991*, pp 95-107.

[18] Stickel M.E.: Rationale and methods for abductive reasoning in natural-language interpretation, in: R. Studer (ed.), *Natural Language and Logic, Proceedings of the International Scientific Symposium*, Hamburg, Germany, Lecture Notes in Artificial Intelligence **459** (Springer-Verlag, Berlin, 1990) 233–252.

[19] Schank, R.C.: *Dynamic Memory: A Theory of Reminding and Learning in Computers and People* (Cambridge University Press, London, 1982).

[20] Winston, P.H.: Learning Principles from Precedents and exercises, *Artificial Intelligence*, Vol. 19, No. 3 (1982).

Appendix

Proposition 1.
If $K \mid\!\sim^{\mathcal{A}} G$ and $K,G \mid\!\sim^{\mathcal{A}}_{*} H$, then $K \mid\!\sim^{\mathcal{A}} H$.

Proof of Proposition 1.
For any formula G, if $K \mid\!\sim^{\mathcal{A}} G$ and $K,G \mid\!\sim^{\mathcal{A}}_{*} H$, we write $K \mid\!\sim^{\mathcal{A}}_{m} H$.

i) Subsuming deduction:
if $\mathcal{A}, K \vdash H$ then $K \mid\!\sim^{\mathcal{A}}_{m} H$.
(proof)
$K \mid\!\sim^{\mathcal{A}} K$. (from subsuming deduction of "$\mid\!\sim^{\mathcal{A}}$")
$\mathcal{A}, K \vdash H \Rightarrow K \mid\!\sim^{\mathcal{A}}_{*} H$. (from Definition 3 i))
Therefore, $K \mid\!\sim^{\mathcal{A}}_{m} H$.

ii) Deductive usefulness:
if $K \mid\!\sim^{\mathcal{A}}_{m} H$ and $\mathcal{A}, K, H \vdash L$. then $K \mid\!\sim^{\mathcal{A}}_{m} L$.
(proof)
$\mathcal{A}, K, H \vdash L \Leftrightarrow \mathcal{A} \vdash K \wedge H \supset L$
For any formula G s.t. $K \mid\!\sim^{\mathcal{A}} G$ and $K,G \mid\!\sim^{\mathcal{A}}_{*} H$.

case-i) $\mathcal{A}, K, G \vdash H$ (from $K, G \mid\!\sim^{\mathcal{A}}_{*} H$)
From the premises, $\mathcal{A}, K, G \vdash L$.
Therefore, $K, G \mid\!\sim^{\mathcal{A}}_{*} L$. (from Definition 3 i))

case-ii) otherwise, for some minimal set of atomic formulas E s.t. $\mathcal{A}, E \vdash K \wedge G$,
$\mathcal{A}, E \vdash K \wedge H$. (from $K, G \mid\!\sim^{\mathcal{A}}_{*} H$)

Therefore, $\mathcal{A}, E \vdash L$.
Thus, $K, G \mid\!\sim^{\mathcal{A}}_{*} L$.

Thus $K, G \mid\!\sim^{\mathcal{A}}_{m} L$.

iii) Consistency:
if $K \mid\!\sim^{\mathcal{A}}_{m} H$ and $\mathcal{A} \cup K$ is consistent, then $\mathcal{A} \cup K \cup \{H\}$ is consistent.
(proof)
$\mathcal{A} \cup K$ is consistent.
$\Rightarrow$ $\mathcal{A} \cup K \cup \{G\}$ is consistent. (from $K \mid\!\sim^{\mathcal{A}} G$)
$\Rightarrow$ $\mathcal{A} \cup E$ is consistent. (from $K, G \mid\!\sim^{\mathcal{A}}_{*} H$)
$\Rightarrow$ $\mathcal{A} \cup K \cup \{H\}$. (because $\mathcal{A}, E \vdash K \wedge H$)

Corollary 1.
If $K \mid\!\sim^{\mathcal{A}}_{*} G$, then $K \mid\!\sim^{\mathcal{A}} G$.

Proof of Corollary 1.
$K \mid\!\sim^{\mathcal{A}}_{*} K$ (from subsuming deduction)
If $K \mid\!\sim^{\mathcal{A}} K$ and $K, K \mid\!\sim^{\mathcal{A}}_{*} G$, then $K \mid\!\sim^{\mathcal{A}} G$. (from Proposition 1)
Therefore,
If $K \mid\!\sim^{\mathcal{A}}_{*} G$, then $K \mid\!\sim^{\mathcal{A}} G$.