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A SYMBOLIC FRAMEWORK FOR QUALITATIVE KINEMATICS

Boi Faltings
Laboratoire d'Intelligence Artificielle, Departement d'Informatique
Swiss Federal Institute of Technology (EPFL)
MA-Ecublens, 1015 Lausanne, Switzerland
Tel. +41-21-693-2738, E-Mail: faltings@CLSEPF51.BITNET

Abstract An important prerequisite for the application of Artificial Intelligence techniques to problems in the physical world is symbolic modelling of the continuous physical systems, the central topic of research in *qualitative physics*. An especially difficult class of such problems is spatial reasoning, as the continuum is multidimensional. This paper presents a symbolic approach to the analysis of mechanism kinematics, an important example of spatial reasoning.

This paper first reviews the theory of *place vocabularies*, which provides a symbolic qualitative representation of mechanism kinematics. The place vocabulary defines the requirements for the spatial reasoning processes used for its computation. This computation can be split into two parts: symbolic *reasoning* and access to *metric dimensions*. The symbolic part defines the conditions that decide the possible device behavior in the form of predicates. The predicates are evaluated by an abstract access procedure which refers to the *metric diagram*. This paper shows how this combination of bottom-up symbolic processing and top-down metric measurements mirrors human problem-solving behavior. The computation model is applicable to other domains as well.

As a third part, the paper shows how the conditions defined in symbolic processing can be used to solve problems of mechanical design, and discusses the integration with existing frameworks of qualitative physics. It also gives an existance proof that purely symbolic spatial reasoning is possible.

1 Introduction

An important problem in Artificial Intelligence is spatial reasoning about physical objects. A solution to this problem has long eluded researchers and its lack is a major obstacle to the application of AI to problems in the physical world. Important examples of spatial reasoning problems are encountered when analyzing mechanism kinematics. For example, understanding the function of a ratchet, shown in Figure 1, requires sophisticated spatial reasoning about the interactions of its parts. The ratchet consists of two parts, a wheel and a lever, both hinged at a fixed axis. The geometries of these parts restrict their relative motion and achieve the desired behavior of the ratchet. The goal of *qualitative kinematics* is a qualitative description of behavior based on the geometries of parts[1].

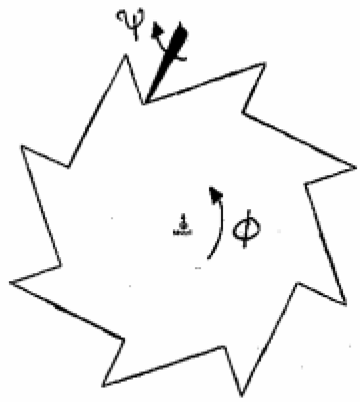

Figure 1: *A ratchet.*

While there exist numerical methods for the simulation and analysis of such devices, reasoning about them in a *symbolic* framework has so far not been adressed. Symbolic modelling of continuous physical processes is the subject of *qualitative physics* ([HAY79,DKL84,FOR84]). In most theories of qualitative physics, the continuous quantities of physical systems are represented by intervals of real numbers. However, such interval-based reasoning can not be directly generalized to the inherently multi-dimensional problem of kinematic analysis.

In earlier work ([FALT86,FALT87a,FALT87b,FNF87,

[1] A qualitative description of the ratchet's behavior is given in Section 1.2

FALT88a]), we have developed the theory of *place vocabularies* for qualitative reasoning about mechanism kinematics. A mechanism's place vocabulary is a graph that represents the set of its possible kinematic states and transitions between them. It forms the spatial substrate for computing an *envisionment* of the mechanism's actual dynamic behavior. Currently, the place vocabulary theory is restricted to the analysis of two-dimensional higher kinematic pairs ([FALT87a,FALT87b,REU75,REU76]). A higher kinematic pair is a pair of two objects, each hinged so that they have only a single degree of freedom. All results described in this paper are for this restricted domain; their generalization has not yet been investigated. In this paper, we first give a brief review of the place vocabulary theory. We then present the model for its computation, based on a metric diagram. Finally, we discuss its integration with qualitative physics theories and applications to spatial problem solving.

1.1 Review of the Place Vocabulary Theory

A complete representation of the behavior of a device is a prerequisite to reasoning about it. Classical numerical analysis techniques are only capable of producing examples of behavior at certain numerical values, not the required complete description. A *qualitative* representation describes intervals of continuous parameters by the same qualitative value. The complete covering of the ranges of each parameter by qualitative values provides the required complete behavior description.

The kinematic properties of physical objects depend on the sizes of shape features in highly nonlinear ways. A qualitative representation of the shapes themselves thus gives very little information about the possible kinematic interactions. Instead, the qualitative description must be introduced at the level of the possible motions of the objects. In the place vocabulary representation, we transform the problem into *configuration space*, and compute a qualitative representation in this domain. This representation consists of a set of regions of configuration space, called places, arranged in a graph representing their adjacencies. It is similar to the place vocabularies first used in FROB ([FOR81]).

The position of a physical object can be described by a small set of parameters. In the case of unrestricted motion in three dimensions, there are three Euclidian position parameters and three orientation parameters which completely determine the placement of the object. In most cases, a single parameter is sufficient to describe the position of a mechanism part, since its freedom is restricted by joints. We call the space spanned by the parameters characterizing the positions of all the objects of a mechanism its *configuration space* ([LPW79,LMT83,DON84]). At any time, the position of all the parts of the mechanism corresponds to a particular point in this space, which we call a *configuration*. As the parts of a mechanism mutually constrain their positions, the configuration space consists of regions corresponding to legal and illegal configurations. We call the union of all legal regions the *free space* and its complement the *blocked space*.

The configuration space for the complete mechanism is formed as the product space of the configuration spaces for all its parts. Each point in this configuration space specifies the position of all parts of the mechanism, and so the dimensionality of this configuration space can become very large. However, in a pratical mechanism each part only interacts with a small number of others, and its freedom is restricted by joints. Specifically, a mechanism is a *kinematic chain* of *kinematic pairs* ([REU75,REU76]). A kinematic pair is a pair of interacting parts. We distinguish *lower pairs*, in which the same type of contact between the parts is maintained throughout their possible motion, and *higher pairs*, in which the contact varies. As shown in [FALT87a,FALT87b,FALT88a]), the behavior of a kinematic chain can be analyzed by composition of analyses for kinematic pairs, and place vocabularies need only be computed for each kinematic pair.

As there are only six different possible lower pairs ([REU75, REU76]), their symbolic analysis is straightforward. Most of the interresting functions in mechanisms are performed by higher pairs, such as escapements, ratchets and gears. There are an infinite number of higher pairs, and we restrict our theory to the analysis of higher pairs only. An important characteristic of higher pairs in mechanisms is that both parts are restricted to one degree of freedom each, either rotation or translation.

Configurations on the boundary between free and blocked space are characterized by the fact that points in both blocked and free space can be reached by an infinitesimally small motion. This is the case only if the 2 objects touch. Pairs of objects can touch in one of three ways: the touch may be between a pair of boundary segments, a vertex and a boundary segment, or a pair of vertices. A configuration which satisfies the last condition is called a *touchpoint* and is already subsumed by the case of a vertex touching a boundary segment. The other two types of touch give rise to *configuration space constraints*. We call the constraints generated by a vertex touching a boundary *vertex constraints* and the ones generated by the touch of two boundary segments *boundary constraints*. Boundary and vertex constraints cover all possible cases of touch and are therefore the only possible boundaries between free and blocked space. The actual boundaries between regions of free and blocked space are defined by the envelope of the constraints. Constraints are subsumed (and not applicable) wherever they fall inside this envelope.

The way that the parts of the mechanism touch each other defines how motion and forces are transmitted. This configuration of contact forms the basis for the dynamic analysis of the device, and must be expressed in the place vocabulary. Note that the configuration space constraints themselves are defined by different points of contact. Thus, the configuration of contact in some particular arrangement of the objects is given by the constraints that are satisfied. Each different physically satisfiable combination of constraints defines a *place*.

In order to reason about kinematic interactions, we require the qualitative relations between the motion parameters of the objects to be constant within each place. Such a relation exists only if the two objects are in contact, this is the case if the place is a constraint segment. The relation is then given by the derivative of the constraint in a coordinate system made up of the two parameters and is qualitatively constant whenever the constraint is *monotone* in the coordinate system. The condition of montonicity sometimes requires further subdivisions of the places, as discussed later in this paper.

It has been shown ([MAS79,ERD84]) that forces and moments in physical space are equivalent to forces on the corresponding point in configuration space and can be analyzed in configuration space. The configuration space formalization is sufficient for their analysis - no reference to the original object shapes is required for the dynamic analysis. The dynamic analysis of mechanisms based on the place vocabulary is the topic of current research by Paul Nielsen ([NIE88]), and the reader is referred to his work for a more detailed analysis. A detailed discussion of place vocabularies can be found in earlier papers ([FALT88a,FALT87a,FALT87b,FALT86]).

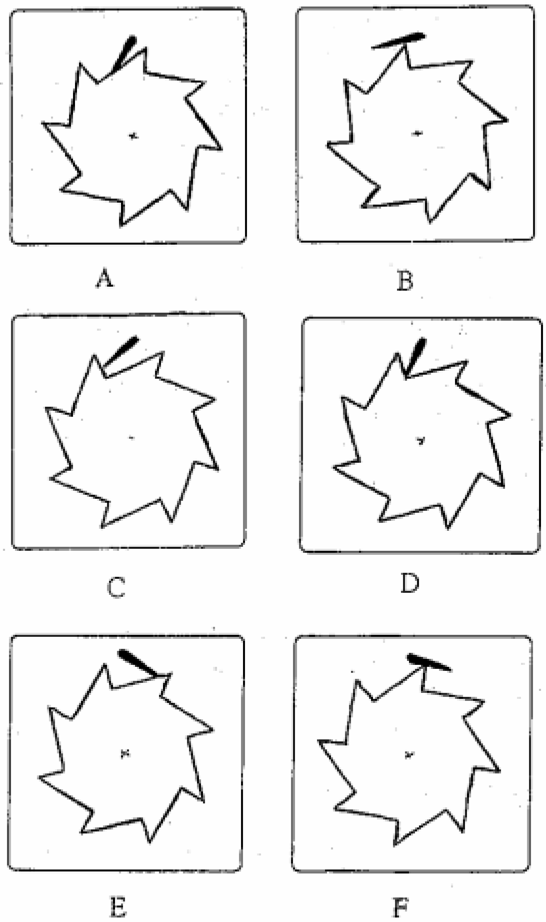

Figure 2: *Examples of possible states of the ratchet.*

1.2 Example of a Place Vocabulary

As an example of a place vocabulary, consider how it can be used to express the behavior of the ratchet example. An explanation based on sequences of kinematic states, such as shown in Figure 2, is natural and intuitive to people. The kinematic states shown in Figure 2 are elements of the ratchet's place vocabulary, augmented by specifying the directions of motion of the elements.

Under the influence of gravity, the lever is pulled toward the position where it hangs straight down. We find that there are two stable states where the lever points either to the left or to the right, supported by the wheel. First, consider the behavior when the wheel turns counterclockwise. When the lever points to the right, we find the sequence of states e) → f) → e) → ..., repeated for each tooth. When the lever points to the left, the behavior of the ratchet is characterized by the sequence of states a) → b) → c) →

a) → ..., also repeated for each tooth. Now, consider the case where the wheel turns clockwise. With the lever pointing to the right, the sequence of states is f) → e) → f) → ..., the reverse of the counterclockwise sequence. However, when the lever points to the left, the sequence of motions is b) → a) → d). From state d), there is no further transition in which the wheel could turn clockwise; the ratchet is blocked. When the lever is in a configuration where it points to the left, the ratchet thus blocks turning the wheel in a clockwise direction, but does not affect its motion in the counterclockwise direction. This is the function of the ratchet.

Note that the place vocabulary expresses *all* the motions allowed by the objects' shapes. It is the spatial component of a subsequent dynamic analysis, which takes into account the Newtonian mechanics of the device. The ratchet place vocabulary thus allows transitions between states a) and e), for example, which must be ruled out for the ratchet to function properly. This is achieved by external forces, and not known until the dynamic analysis.

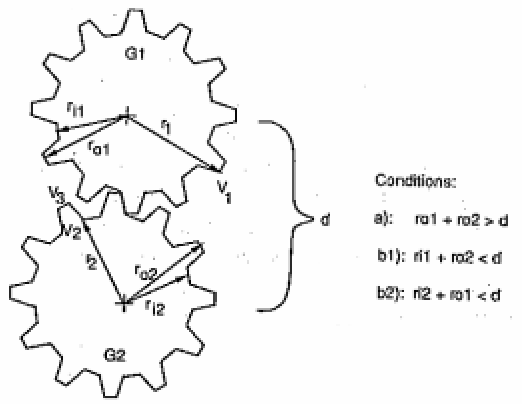

Figure 3: *A pair of gears meshes only when the distance d between their centers is a) small enough for the gears to touch, and b) large enough so that the teeth fit into each other. There are other conditions as well.*

2 The Metric Diagram Model

Place Vocabularies can be computed in a quite straightforward manner using methods of computational geometry. However, such methods do not allow reasoning about the aspects of the particular shapes that are important for achieving a particular behavior. Human reasoning is *goal-directed* and poorly modeled by the classical bottom-up techniques of numerical analysis. People are very good at determining the effects of changes of particular features on the behavior of a device. For example, when presented with a pair of gearwheels, people can readily state conditions that the distance between their centers must satisfy in order for the gears to mesh, as shown in Figure 3. This type of reasoning is important in many spatial reasoning problems including mechanical design, troubleshooting, learning new physical phenomena, and reasoning under uncertainty.

In this paper, we propose an novel computation model in which metric information is used in a top-down manner. The physical objects are described in a *metric diagram.* The metric diagram, an example of which is shown in Figure 4, can be broken into two parts:

- a symbolic description of the object features and their adjacencies
- the numerical values of the dimensions of the shape features, which may be fully or only partially known

The symbolic part is a bottom-up description of the scenario, very similar to a primal sketch obtained from

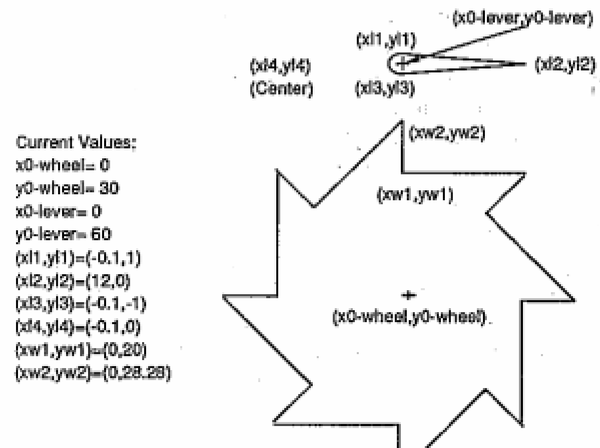

Figure 4: *The metric diagram for the ratchet example. The symbolic part describes the adjacencies of the object features, while the metric part consists of the actual values of the parameters.*

a vision system. It defines a *language* in which quantities describing the metric dimensions of objects can be defined. The simplest of these quantities are distances between vertices defined in the symbolic description. The language also allows more complex definitions such as the maximum width of an object orthogonal to a line between two vertices, or the maximum overall width of an object. By reasoning from the symbolic description, the program finds the conditions on the values of the metric dimensions of the objects that determine their kinematic behavior. These conditions are expressed as *predicates* on the sign of algebraic expressions made up of quantities defined by the language of the metric diagram.

The evaluation of the conditions is the task of an *access procedure*, an abstract oracle which we assume capable of evaluating arbitrary algebraic conditions. It may also return an ambiguous value is not enough information is known. In our implementation, the access procedure consists of tests on exact metric representations of objects. In general, it may be implemented by a combination of internal and external processes. Internal implementations use only information that is already represented in the system. For example, the sizes of the objects may be globally categorized in some interval-based system, and this information may be sufficient for many predicate evaluations. External implementations are responsible for measurements to obtain missing information. For example, distances can be compared by placing objects against each other, or by drawing them on paper.

The idea of referring to an explicit diagram to decide on conditions in a top-down manner originated in FROB ([FOR81]) and was further elaborated in [FNF87]. In this earlier work, the metric diagram was used to perform tests that help the dynamic analysis. Because using such tests contradicts the completeness of qualitative representations, their use is very limited. The explicit definition of algebraic conditions on *static* parameters presented here points out an entirely different way of using the metric diagram.

2.1 The Metric Diagram and Human Problem Solving

The metric diagram access procedure models human problem solving behavior in the mechanism domain. When a certain predicate is very clearly decided by the metric dimensions, it can be evaluated by an internal implementation of the access procedure. In this case, people consider the resulting structure as "obvious" and may even fail to notice the existance of a condition. For example, in the ratchet example shown in Figure 1, the angle at the tip of the lever has to be smaller than the opening angle of the teeth, for otherwise the lever will not fit between the teeth properly. This condition is so obviously satisfied that it is very hard for the human observer to even notice it. On the other hand, in Figure 1 it is very hard to tell if there is sufficient distance between the lever and the wheel to allow it to pass under the lever at all. The reader may feel the urge to take additional measurements on the objects. This corresponds to using an external implementation of the access procedure.

2.2 The Metric Diagram in Qualitative Kinematics

The conditions on the metric parameters that arise in the computation of place vocabularies are described in detail in [FALT87b,FALT88b]. In this paper, we can only give an example of the predicates that the access procedure evaluates during the place vocabulary computation.

Consider the two gears $\mathcal{G}_1$ and $\mathcal{G}_2$ in Figure 3. A particular pair of vertices $\mathcal{V}_1$ on $\mathcal{G}_1$ and $\mathcal{V}_2$ on $\mathcal{G}_2$ can touch only if their distances from the centers of rotation, r_1 and r_2 satisfy the following relation with respect to the distance d between the two centers of rotation:

$$d < r_1 + r_2 \ and \ d > |r_1 - r_2|$$

Similarly, a touch between a $\mathcal{V}_1$ and the boundary between $\mathcal{V}_2$ and $\mathcal{V}_3$ is only possible if there exists at least one point on this boundary which satisfies the above condition. In this case, this can be expressed by the condition that at least one of $\mathcal{V}_2$ and $\mathcal{V}_3$ satisfies the condition for a possible touch with $\mathcal{V}_1$.

This distance criterion is not the only condition that arises in the place vocabulary computation. Other conditions exist for the existance of subsumptions between different types of contact, and for the existance of changes in qualitative relations between parameters.

3 Kinematics and Qualitative Reasoning

In this section, we show how using the metric diagram allows us to integrate qualitative kinematics with the principles of established qualitative reasoning theories, and illustrate possible applications, such as mechanical design and variable modelling. For the purposes of this discussion, we treat the place vocabulary as a complete specification of the mechanism kinematics. A complete envisionment of a mechanism's actual behavior has to take into account forces on the objects and is obtained by a dynamic analysis based on the place vocabulary. This is the topic of current research by Paul Nielsen ([NIE88]).

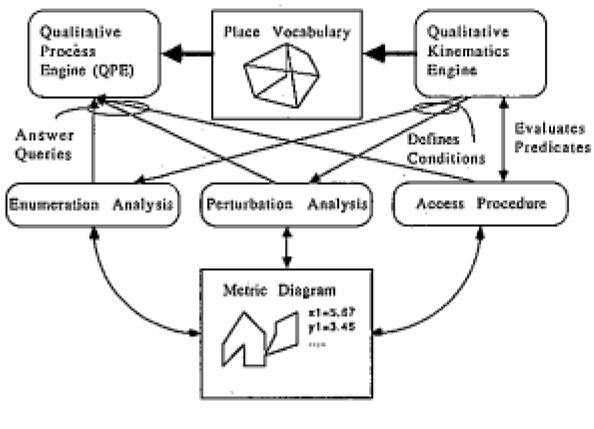

Figure 5: *System Architecture. The Qualitative Kinematics Engine provides the place vocabulary for dynamic analysis by the Qualitative Process Engine. It refers to the metric diagram via the access procedure, and supplies information to enumeration and perturbation analysis modules. The Qualitative Process Engine can reason about static parameters via all three modules.*

3.1 Integration with Qualitative Physics Theories

The distinction we have made between symbolic and metric information can be applied to most qualitative reasoning problems. In many domains, such as circuit analysis, the symbolic information defines the situation precisely enough so that quite accurate predictions of behavior can be made without information about the metric parameters. Qualitative reasoning methodologies, such as qualitative process theory ([FOR84]), allow the specification of *quantity conditions* on metric parameters to decide between ambiguous predictions. The predicates in our computation model could be stated as quantity conditions in such a framework. However, the dimensions of the objects do not change as part of a mechanism's behavior and should therefore be considered different from the dynamic parameters such as forces and positions. Using the metric diagram access procedure provides this separation by hiding the static parameters from the high-level analysis. In order to allow spatial problem solving, additional mechanisms which allow reasoning about variation of static parameters must be provided. This is the function of the perturbation and enumeration analysis, described in this section. The resulting architecture is shown in Figure 5.

Note that making each metric predicate a quantity condition would in principle allow reasoning about kinematics without *any* knowledge of the metric dimensions of the objects, with the unknown quantity conditions causing ambiguities. This is an existance proof of a purely symbolic kinematics. We use the word "symbolic" instead of "qualitative" because the predicates may contain complicated algebraic expressions of quantities. The stronger classification "qualitative" should be reserved for systems that use only inequalities between observable quantities themselves.

3.2 Perturbation Analysis

The prediction of the effects of changing a parameter in a given device is an important problem, particularly in problem-solving applications, where the behavior of a device is to be modified by parameter changes. We call the analysis to find a suitable change *perturbation analysis* of the device. In the place vocabulary computation, we mark each element of the place vocabulary with all the predicates whose value has contributed to its existance and particular form. The set of parameters that can influence the element is given as the set of parameters whose values were used in the computation of the predicates. It is left to the application using the place vocabulary to decide which parameter should be varied, as this must be carried out by domain-dependent heuristics.

When a parameter to vary is picked, the system is faced with the problem of determining what new value it should be changed to. To determine a suitable value, we find the *landmark values* of the parameter where the predicates under consideration change their values. As each predicate is defined as the test of the sign of an algebraic expression, the roots of this expression in the parameter define the desired landmark values. To change the behavior, the parameter value must be changed beyond this landmark value.

As an example, consider the ratchet introduced earlier. The metric diagram, shown in Figure 4, defines the metric parameters of the device. The particular choice of values in Figure 4 results in a place vocabulary containing a situation where the lever can push the wheel in the clockwise direction, as shown in Figure 6. If the wheel is connected to a transmission of gears, this may cause rattle and unnecessary wear. Suppose that we would like to modify the design to eliminate this behavior.

The state shown in Figure 6 is represented as a place in the place vocabulary. In the computation, it has been marked with two conditions for its existance. The first condition is for the existance of this type of contact itself and turns out not to be useful for our purposes, as changing its value also renders the ratchet disfunctional. The second condition is dependent on the following parameters: X0-WHEEL, X0-LEVER, Y0-WHEEL, Y0-LEVER, XW2, YW2, XL2, YL2, XL3 and YL3. One way to fix the design is to shorten the lever, this corresponds to varying the parameter XL2. The program transforms the condition into

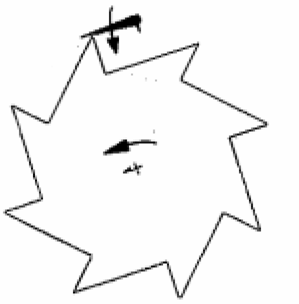

Figure 6: *In this state, pushing down on the lever pushes the wheel in a clockwise direction.*

```
(places::explore-condition cseg-17 T)
Element CSEG-17 has conditions for (EXIST)
choose one by number: 0

There are 2 conditions, choose one by number: 1

The expression contains the following variables:
(Y0-WHEEL Y0-LEVER X0-WHEEL X0-LEVER YW2 XW2 XL3 YL3 YL2 XL2)
select one: xl2

The condition is

 (+ (ABS XL2)
   (* -1
      (SQRT (/ (+ (* -56.568541999999994d0 (ABS XL2))
                  (* 100.0000139986592ld0
                     (SQRT (+ 1.01d0 (* 0.2d0 XL2) (* XL2 XL2)))))
                (SQRT (+ 1.01d0 (* 0.2d0 XL2) (* XL2 XL2))))))) >0

 Variable XL2 has the value 12.
Lower bound on variable variation?: 0
Upper bound on variable variation?: 20
**Interval for XL2 actually used: 0.0f0 .. 20.0f0

 ----landmark-value found in interval 0..20 is 6.697549261152744d0
```

Figure 7: *The program output for analyzing variation of XL2.*

an expression in XL2, as shown in Figure 7. The landmark value found for XL2 is 6.6975, exactly the maximum value for XL2 that will eliminate the undesirable state. If we had chosen to vary the position of the center of rotation of the lever, we would have obtained a landmark value at 61.62136. If we had chosen the height of the teeth of the wheel and varied YW2, we would have obtained a landmark value at 26.5248. By similar analysis, the program can also determine what perturbations might allow us to add certain elements to the place vocabulary, such as to allow the contact between vertex (XL1, YL1) and the tip of the wheel, (XW2, YW2), or to again add the just removed state to the place vocabulary.

However, note that if we actually followed the program's recommendation and made XL2 smaller than the landmark value, we end up with a non-functional ratchet, as the lever would no longer be long enough to block rotation of the wheel. This illustrates the basic weakness of the perturbation analysis: it considers all the parameters only a single condition at a time. In the next section, we introduce a complementary analysis, which determines the effects on all the conditions of varying a single parameter.

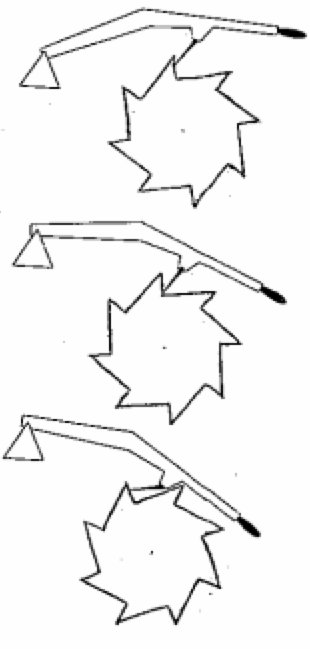

Figure 8: *The ratchet is either completely blocked, engaged or disengaged, depending on the position of the large lever.*

3.3 Enumeration Analysis

For a given parameter, the set of predicates in which it occurs defines a complete set of landmark values. A change in a parameter can influence the place vocabulary only if it passes one of the landmark values of the parameter. The landmark values can be ordered on the real axis, so that within each interval between two landmark values, the place vocabulary is the same for all values of the parameter. By computing the place vocabulary for a representative value of the parameter in each interval, we can find a complete list of all possible place vocabularies that can be achieved by varying the parameter. We call the computation of such a list an *enumeration analysis*.

In design problems, enumeration analysis is useful when the perturbation analysis has indicated which parameter is to be varied and it must be determined whether its variation also causes other, perhaps undesirable, changes. It is also necessary for the analysis of mechanisms with switches, such as the device shown in Figure 8. The position of the large lever varies the qualitative behavior of the device. As the large lever stays stationary during the normal movement of the mechanism, its position is not a true dynamic parameter of the device. Instead, the different dynamic behaviors allowed by the different positions can be represented by a graph of models ([PEN87,ADA87]). By enumeration analysis, we can find the distinct models in this graph and the conditions for their selection.

In this example, we vary the distance between the centers of rotation of the wheel and the ratchet lever, a variation of Y0-LEVER in the metric diagram (Figure 4). The enumeration analysis of our program finds a total of 14 landmark values, at 51.0045, 52.479, 53.1375, 58.30565, 59.0998, 59.2725, 59.28925, 59.38427, 60.72458, 61.62136, 62.0, 67.7359 and 70.28427. The same landmark values exist for choices of Y0-LEVER below the wheel, but are not considered because only an interval of 30 to 80 was given for the binary searches that determine the landmark values. Importantly, the fact that only fourteen landmark values exist shows that the approach is practically feasible.

Below the landmark value at 51.0045, there exist no legal configurations of the objects, and beyond the landmark value at 70.28427, no contact between the objects is possible. The most significant changes in behavior occur at 59.38427, where it becomes possible for the wheel to turn, and at 62.0, where the wheel can turn freely in both directions. Choices of Y0-LEVER in the interval between 59.38427 and 62.0 result in a functioning ratchet. The other landmark values reflect changes in possible contact relationships and breakups of places.

The enumeration analysis thus provides us all possible models that can be achieved by varying the position of the large lever in Figure 8. It also shows us a proper solution to the problem we investigated in the previous section: the undesirable state could be eliminated by choosing Y0-LEVER at a value greater than 61.62136, and a choice between 61.62136 and 62.0 will give us the desired ratchet design.

In most applications of qualitative kinematics, there will be several unknown parameters, with their values usually restricted by external constraints. For example, if we are designing a ratchet, we are usually not free to choose any conceivable size for the parts. They may not be larger than the space provided, but must be large enough to support attaching other parts. The parameters of the design are therefore not entirely unknown, but merely uncertain. They are thus best handled by the perturbation technique. However, the problems of actually picking a proper parameter value, as well as those of variation of the modeling of a device seem to require some sort of enumeration analysis. A practical application must be based on a combination of the two techniques.

4 Conclusions

In this paper, we have presented two major new results. First, we have introduced the place vocabulary theory for qualitative kinematics, an important problem of spatial reasoning. We have shown how the theory allows the analysis of mechanism kinematics in a purely symbolic and logical framework with very limited numerical computation. We have then shown a particular model for the computation of place vocabularies which separates bottom-up symbolic processing and top-down metric measurements based on a metric diagram. We have argued that this model mirrors human problem-solving behavior, and allows symbolic reasoning about the variations of dimensions of the mechanism parts.

It is very important to note the differences between a qualitative analysis as presented here and a numerical analysis of kinematics. The numerical analysis relies on the number system and is thus more efficient than a symbolic analysis. However, it is fundamentally incomplete, because behavior can only be predicted at *points* in space-time. The symbolic description, while harder to compute and less "precise", is complete and never fails to predict a possible behavior. Being less "precise", it can also be computed with incomplete information about the device. A third advantage is that it is immediately comprehensible to people and requires no further interpretation.

The system architecture was inspired by observation of human problem-solving behavior. People are very good at stating the conditions under which a certain behavior is possible. For example, when we predict the effect of changes in the shapes of mechanism parts, we seem to use these conditions to determine landmark values where the variation changes the behavior. Similar to the prediction of our model, we are not very good at predicting what happens when many parameters are changed simultaneously.

Similar to the solution of kinematics problems, other problems involving symbolic reasoning about continuous physical processes can also be solved using a metric diagram. As a computation model, it combines the bottom-up computation of a symbolic *description* of the world with top-down *measurements* of quantities defined by the description. A division of the computation into these two types of processes is useful for the solution of many problems involving perception and metric measurements. As our research focusses on spatial reasoning, we have not yet investigated other applications of the model.

4.1 Acknowledgements

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