PERFORMANCE ESTIMATES FOR THE DADO MACHINE:
A COMPARISON OF TREAT AND RETE*

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ABSTRACT

DADO is a highly parallel, VLSI-based, tree-structured computer, intended for the rapid execution of production system programs. In this paper we describe a new match algorithm for executing production systems on DADO that is capable of handling both temporally redundant and nontemporally redundant production systems. We argue that the new algorithm is faster than the original DADO algorithm intended for nontemporally redundant systems. We also show that the expected performance of the new algorithm executed on parallel hardware is faster and more space efficient than parallel implementations of the RETE match algorithm, which is appropriate for temporally redundant systems.

1 INTRODUCTION

DADO is a highly parallel, VLSI based computer comprising a large number of processing elements (PE's) interconnected in a complete binary tree. Adjacent to the root of the DADO tree is a conventional coprocessor which acts as a file server and performs the usual activities of a host. Thus, DADO may be viewed as a peripheral device of a conventional machine. Communication between PE's in the DADO machine occurs along the tree edges. In addition, any PE in the DADO machine may broadcast data to all of its (logically) connected descendants in the tree, or may be instructed to report a value to all of its ancestors.

In the DADO prototype now operational at Columbia, there are 15 PE's (Stefo et al., 1983). Each PE is composed of an 8 bit processor, a ROM resident operating system, 8K bytes of RAM, and an I/O section. The DADO2 prototype presently under construction has 16K bytes of RAM at each of 1023 PE's. Under the control of software, a PE may operate in one of two modes: master or slave. In master mode the PE runs a computer program stored in its local memory. However, instructions embedded within the master's program may be broadcast to descendant PE's operating in slave mode for immediate execution. Each of the slave PE's executes the instruction on different data stored in its local RAM in a manner similar to an array processor, or the ILLIAC IV (Lowrie, 1975). This type of parallelism is known as single instruction stream multiple data stream (SIMD) execution (Flynn, 1972). Furthermore, the machine can be arbitrarily partitioned into a number of independent subtrees. The root of such a subtree logically disconnects itself from its parent and becomes the master of the PE's logically connected below. This type of machine has become known as a multiple SIMD (MSIMD) architecture (Siegel, 1981).

The DADO machine has been designed as a special purpose processor capable of achieving significant performance improvements in the execution of production system programs.

A production system (PS) (Newell, 1973) is defined by a set of rules (or productions) and a collection of dynamically changing facts called the working memory (WM). A rule in a production system consists of a left hand side (LHS) and a right hand side (RHS). The LHS is a collection of condition elements to be matched against the contents of the WM. The RHS contains actions effecting changes in the WM. A production system repeatedly executes the following cycle of operations:

1. Match: For each rule, compare the LHS against the current WM. Determine if the WM satisfies the LHS.

2. Select: The set of satisfied rules is called the conflict set. Some subset of the conflict set is chosen according to some predefined criteria.
3. Act: Add to or delete elements from the WM as specified by the RHS of the selected rules.

An example rule using the OPS5 production system language (Forgy, 1981) assumed to be familiar to the reader is shown in figure 1. The pair of WM-elements matching the condition elements is called an instantiation of the rule.

```
(p categorize-job-sizes ; rule name
  (message 'job <x> size <y> status new)
  ; <x> and <y>

  (class-def 'size <y> 'class-name medium)

  (make job 'job-name <x> 'class-name medium)
  )
```

This rule may be read as:

If there is a WM-element in the system representing a message about a new job, and the job's size matches the class definition for medium size jobs, then create a new WM-element tagging the job with the class name medium.

Figure 1: An Example Production Rule.

1.1 Temporal Redundancy

A distinguishing property of production systems is temporal redundancy. A PE is considered temporally redundant if on each cycle few changes to WM are made. R1/XSEL, which incrementally builds a solution to the VAX configuration problem (see McDermott, 1982), is typical of temporally redundant PS's. Systems which search through large databases, such as ACE (Stolfo and Vescan, 1982), or sensor based systems as would be found in a robot, tend to be nontemporally redundant.

2 THE ORIGINAL DADO ALGORITHM

One approach to the parallel execution of PS's on the DADO computer is to logically divide the tree into three distinct components. One of these components consists of all PE's at a particular level within the tree, called the PM-level. The PM-level delimits an upper and lower portion of the tree (see figure 2).

A fine grain DADO computer would be one with perhaps a hundred thousand very simple PE's, each having 1 to 2K bytes of memory. For a fine grain DADO, the original DADO algorithm (Stolfo and Shaw, 1982) suggests each PE at the PM-level be used to store a single production. The portion of working memory relevant to the rule contained in the PM-level PE is distributed uniformly in the subtree below. A working memory element is considered relevant if it satisfies the constants and the intra-condition (Forgy, 1982) restrictions of any condition element in the rule. The subtrees formed by the PM-level are to be considered as a collection of independent parallel associative processors (Foster, 1978) providing parallel access to the WM-elements.

The upper portion of the tree is used for synchronization and selection. The original algorithm is described as follows:

During the act portion of the production system cycle, additions to WM are performed by broadcasting the WM-element to all the PE's in the tree. The PM-level PE's determine if the WM-element is relevant to their rule. If so, the WM-element is stored in an available PE. Deletions from WM are processed by broadcasting the WM-element to all PE's in the tree. The PE's then compare the broadcast element to the element in their local store. If it is the same, the PE marks itself as free.

During the match phase, each PM-level PE enters master mode and broadcasts to its slave PE's the first condition element of the rule. The slave PE's compare the pattern against the working memory elements and report, to the PM-level master, the success of the match and the bindings of any variables in the condition element. Each variable binding is then substituted in the remaining condition elements, and the match routine is called recursively for the remaining condition elements.

Since no state is saved between cycles, and the algorithm exploits massive parallelism during the match phase, the original algorithm is considered to be best suited for nontemporally redundant production systems where many changes to WM occur on each cycle of execution.

3 THE RETE MATCH

A medium grain implementation of DADO as opposed to a fine grain implementation would comprise on the order of tens of thousands of PE's, each made up of a state-of-
The RETE algorithm first compiles into the network sequences of tests which perform partial matches of condition elements. These tests are called single input tests since they consider only one attribute of a condition element and one token at a time. Thus, each node has only a single arc entering and leaving the node. The match network for the rule in Figure 1 is shown in Figure 4.

![RETE Match Network](image)

Figure 4: RETE Match Network for the Rule in Figure 1.

Associated with the last output arc of a chain of one input tests is a token memory node called an alpha-memory. Plus tokens that have satisfied the one input tests are added to the alpha-memory. Note that the alpha-memories contain precisely those WM-elements that are relevant to a particular condition element. Minus tokens that have reached an alpha-memory node have a corresponding plus token already present in the alpha-memory. The corresponding plus token is removed. Both types of tokens propagate further to two input test nodes.

The two input test nodes test for consistent variable bindings between two condition elements. When a token enters a two input node, it is compared against the tokens in the memory on the opposite arc. Any tokens which have consistent variable bindings are paired with the first token, to form a new token, that is stored in a token memory and propagates to the next node in the network. Token memories that store paired sets of tokens...
are called beta-memories. Tokens that propagate from the last two input nodes reflect changes to the conflict set.

4 THE TREAT ALGORITHM

The original DADO algorithm does not save any state across production system cycles. In a temporally redundant PS, where few WM changes are made on each cycle, the original algorithm must recompute many comparisons of WM. The opposite is true of the RETE algorithm. The RETE algorithm saves sufficient state in the match network to guarantee that no comparison of two working memory elements is recalculated at a later cycle. If large changes to WM are made, a large overhead is incurred maintaining the state information.

The Temporally REDundant Associative Tree algorithm (TREAT) for production systems on DADO attempts to synergistically merge the advantages of the two aforementioned algorithms. The approach of the TREAT algorithm exploits the observation that most of the state saving effects of the RETE match are achieved by partially matching the condition elements and retaining the conflict set between cycles. In other words, it is important to construct the alpha-memories and to remember the conflict set between cycles, but the beta-memories are of little use. We will argue below that state saved by the construction of beta-memories is less beneficial than the overhead involved in their maintenance.

Further, though TREAT may have to recompute some comparisons, there are many processors available to do the computation and the delay required for the computation may be negligible. In a VLSI machine based on an intelligent memory paradigm, the tradeoff between having memory to store all the contents of the beta-memories or having sufficient processors to recompute them quickly could lean towards the latter.

The first observation that leads to the development of TREAT is that when a new element is added to WM, any new rule instantiations entering the conflict set must necessarily contain the new WM-element. Therefore, the new WM-element may be used as a seed which acts as a constraint when building new rule instantiations. By constructing the alpha-memories we can quickly compute the set of condition elements which match the new WM-element. When the match proceeds with the remaining condition elements, only the subset of WM that has partially satisfied each condition element is considered.

By similar reasoning, if a WM-element is removed from WM, then any rule instantiations removed from the conflict set must also contain the WM-element. The TREAT algorithm stores the conflict set in a distributed fashion in the DADO tree. When a WM-element is deleted, the conflict set is examined in a parallel associative manner and all conflict set elements containing the WM-element are removed from the conflict set concurrently.

If rules contained only positive condition elements, the two actions above would be sufficient. When a WM-element matches a negated condition element, the algorithm is slightly more complicated. If the action of a RHS adds a WM-element that matches a negated condition element, then some rule instantiations in the conflict set may have to be removed. Unlike the removal of a WM-element that matches a positive condition element, the WM-element matching the negated condition does not appear explicitly in the conflict set. To determine which conflict set elements must be removed, the condition element is temporarily considered to be positive and the new WM-element is used as a seed to build rule instantiations. These rule instantiations are then compared with the conflict set. Any instantiation appearing in the conflict set is removed.

The fourth case is when a WM-element is removed, and it matches a negated condition element. In this case, removing the element may permit rule instantiations to enter the conflict set. These new rule instantiations are precisely those that would enter if the condition element were positive and the WM-element had just been added.

There may, however, be another WM-element similar to the removed element which prevents these new instantiations from entering the conflict set. Such an element would necessarily satisfy the negated condition element precisely the same way as the removed element, i.e., have all the same constant and variable values as the removed element. Before building the new rule instantiations, WM is quickly scanned for such an element.

5 COMPARING TREAT AND RETE

Both TREAT and RETE may be easily explained by adopting the terminology of relational database theory. If the entire WM is considered as tuples in a relational database and each rule is viewed as a database query, then the partial match of TREAT and single input tests of RETE may be considered as a sequence of select operations. The alpha-memories contain the resulting relations. If two condition elements have a common variable, the set of
finding pairs of WM-elements with consistent variable bindings may be viewed as a join of the corresponding alpha-memories.

In RETE, when a tuple enters from one arc of a two input node, it is compared against all the tuples stored in the memory associated with the other arc. Successful pairs of tuples are placed in the beta-memory. The two input test nodes of RETE incrementally build the partial join results, and thus the beta-memories contain partial join results of the query.

In this context, during the act cycle, the TREAT algorithm places changes to WM in "new" alpha-memories. The match cycle is performed by doing a join reduction with each new alpha-memory and the old alpha-memories corresponding to the remaining condition elements of the same rule. After the reduction, the new alpha-memory is concatenated with the old.

The join reduction may be done in any order. It has been shown in relational database systems (Zloof, 1977) that the best way to process a query of this type is to order the join operations by increasing cardinality of the relations. It is a byproduct of this optimization that permits TREAT to perform well for both temporally redundant and non-temporally redundant systems. If changes to the WM are few on each cycle, the new alpha-memories will contain 1 or 2 tuples. The optimization will then use the new alpha-memory as the seed of its query. If, however, there are large changes to WM, the joins will still be performed in optimum order rather than sequencing through the changes.

We note that TREAT must perform a search when WM-elements are added or deleted. If more than half the WM changes each cycle, the original DADO algorithm may still prove to be better.

To maintain consistent beta-memories, RETE must perform the joins in a fixed order. The order is determined at compile time when no information is available about the constituent relations. Indeed, it is not possible to statically determine the characteristics of the relations (Stollo, 1984). Thus, it is unlikely that RETE performs the joins in optimal order.

5.1 Implementing TREAT and RETE on a Medium Grain DADO

The following is common to both algorithms. It has been noted that in many OPS5 programs, the production level parallelism is between 20 and 30. That is, no more than 30 different rules may be satisfied at a given time. The FM-level is selected at the fifth level of the tree with 32 PE's available. The rules are partitioned among the FM-level PE's. It is assumed that there is a good partitioning algorithm that prevents two rules that may be satisfied simultaneously from being placed in the same partition. (See Ishida and Stollo, 1984 for example.)

Within each partition, the condition elements and associated alpha-memories are numbered uniquely. The select operations for each condition element are distributed among the PE's in the WM-subtree. During the act cycle, changes to WM are broadcast to all PE's, which in parallel perform their local tests. Any successful selection is reported to the PM-level processor and the WM is stored in the appropriate alpha-memory.

The alpha-memories are stored in a distributed fashion in each subtree, indexed by a preassigned number (see Fig 5). An effort is made to place at most one WM-element per alpha-memory in any PE. If this is impossible, the disparity in the number of elements in different PE's is never greater than 1. In the TREAT algorithm, changes to WM are saved in a distinct partition until the match phase. In the RETE match the beta-memories are also numbered and stored in a fashion similar to the alpha-memories.

A join step is performed by broadcasting one tuple of one relation to every PE in a subtree. The PE's then compare the broadcast-tuple to any tuples of the second relation stored in their local memory. In TREAT, if the comparison is successful, the second tuple is reported to the PM-level PE and the query continues in depth-first fashion. In RETE, the second tuple is reported, but the pair of tuples must be assembled and stored in a beta-memory. We summarize TREAT by the abstract algorithm in Figure 6.

![Figure 6: Working Memory Elements Indexed by Condition Element Number.](image_url)
1. Initialize: Distribute the match routine and a partitioned subset of rules to each PM-level PE. Load the partial match tests for each condition element in a PE below the PM-level PE containing the associated rule. Set CHANGES to the initial WM elements.

2. Repeat the following:

3. Act: For each WM change in CHANGES do;
   a. Broadcast the WM change to all PEs.
   b. Each PE performs the partial match tests stored in its local memory.
   c. For each successful partial match test, place the change in the corresponding "new" alpha-memory. Each PM-level PE does this independently of the others.
   d. end do;

   a. For each nonempty "new" alpha-memory do;
   b. Associatively probe the old alpha-memory for elements appearing in the new alpha-memory. Remove them.
   c. Case: If the alpha-memory corresponds to a positive or a negative condition element:
      i. Positive: Associatively probe the conflict set for elements containing elements of the new alpha-memory. Remove them.
      ii. Negative:
         1. Associatively probe the old alpha-memory for elements with the same variable bindings as any in the new alpha-memory. If found, remove the element from the new alpha-memory.
         2. Perform a join reduction, in optimal order, of the new alpha-memory and the old alpha-memories of the same rule.
   d. end do;

   a. For each nonempty "new" alpha-memory do;
   b. Perform a join reduction, in optimal order, of the new alpha-memory and the old alpha-memories of the same rule.
   c. Case: If the alpha-memory corresponds to a positive or a negative condition element:
      i. Positive: Add these new instantiations to the conflict set.
      ii. Negative: Associatively probe the conflict set for each of these new instantiations and remove if found.
   d. end do;

Figure 6: Abstract Algorithm Illustrating TREAT.

5.2 Partitioning Algorithms

The TREAT algorithm provides a simple way to detect active rules and provide information for partitioning algorithms.

In the TREAT algorithm, it is easy to maintain a running count of the size of each of the alpha-memories. For a particular rule, if any of the alpha-memories corresponding to its condition elements are empty, then the rule cannot contribute to the conflict set, and no work is performed for that rule. The rule is considered to be nonactive. The overhead for recognizing active rules is small. When updating the size of an alpha-memory, we need only test for transitions from zero to one and from one to zero. Upon this transition, a test must be made of the other alpha-memories for a rule. If they are nonempty, the rule is added or removed from an active list.

It is this test that provides a mechanism for developing adaptive partitioning algorithms. A good partition algorithm would keep the number of active rules in different partitions the same. If the production system monitor discovers two rules in a partition are active at the same time, the monitor may then pass this information to the partitioning algorithm. The partitioning algorithm may then be careful not to place these two rules in the same partition for the next run.

6 EXPECTED PERFORMANCE OF TREAT FOR OPS5

Using statistics generated by studying OPS5 production systems, (Gupta, 1984a) has detailed performance estimates for a fine grain DADO employing the original DADO algorithm as well as performance estimates of a medium grain DADO employing the RETE match. In this section, we make performance estimates of the TREAT algorithm on OPS5 by elaborating on Gupta's study. It should be noted that the study was based on OPS5. The semantics of OPS5 were restricted to facilitate efficient implementation of the RETE match on a sequential machine. The study is not indicative of the performance of a DADO machine employing a less restrictive language that has the ability to express more parallelism in the problem. Ongoing research aims towards the eventual implementation of a production system formalism we have come to call HerbAI (in honor of Herbert Simon and Allen Newell). HerbAI will permit the expression of parallel constructs not presently capable of OPS style systems.

To make use of the data from the OPS5 study, we must first determine how many more
comparisons TREAT requires than RETE as a result of eliminating beta-memories. Since DADO has many processors matching a broadcast data against their local store in parallel, the basic unit that should be counted is the required number of parallel matches.

The RETE Network Representing an Average Rule.

Since the performance of these algorithms is statistical in nature, we can only make a qualitative statement based on the expected performance of an average case. The average rule in an OPSS system has four condition elements. The RETE match network representing the memories and two input test nodes for such a rule are illustrated in Figure 7.

Let's assume that each alpha-memory contains $n$ WM-elements and that there is uniform probability $p$ that any 2 tuples match. Then beta-relations B1 and B2 will contain $n^2p$, $n^2p$ tuples. If a WM-element partially matching C1 is added to the system, it will be compared against $n$ tuples in alpha$_3$, $n$ of them can be expected to match. These, in turn, must be compared to the $n$ tuples of alpha$_4$. This step requires $np$ comparisons and can be expected to produce $n^p$ results. These results, in turn, must be compared to the $n$ tuples in alpha$_4$.

The total number of comparisons for an element entering C1 is then $n + n^2p + n^p$. If the element partially matches C2, the number of comparisons is the same. If we add one more WM-element with equal probability to the four condition elements and do a similar analysis for elements partially matching C3 and C4, the average expected number of comparisons will be:

$$n + 0.5n^2p + 0.75n^3p^2$$

The analysis for an element partially matching C1 does not make use of the results stored in the beta-memories. Since the TREAT algorithm retains no beta-memories, the number of comparisons required for the TREAT algorithm for a new element partially matching any of C1 through C4 is the same as the RETE algorithm for an element partially matching C1. Note that half of the time, when a new element partially matches C1 or C2, the number of comparisons for the two algorithms is identical. The number of comparisons for TREAT is:

$$n + n^2p + n^3p^2$$

These equations reflect the number of individual comparisons. However, the appropriate measure for the DADO machine is the number of parallel matches. If we assume there is no more than one tuple of a relation in a processor, then the expected number of parallel matches is equal to the number of comparisons divided by $n$. Thus:

Parallel Matches for

RETE: $1 + 0.5np + 0.75n^2p^2$

TREAT: $1 + np + n^2p^2$

The TREAT algorithm is only slightly worse. Asymptotically, the two algorithms perform identically. The average values for $n$ and $p$ derived from six large production systems (Gupta, 1984b) are $n = 26.6$ and $p = 0.039$. In this case, the expected number of parallel matches is 2.33 and 2.98 for RETE and TREAT respectively.

However, this is an "unrealistically" average case. Indeed, for the R1 program Gupta reports an average of 58 WM-elements per alpha-memory with a standard deviation of 61. If we remove the assumption that all alpha-memories contain the same number of tokens, what is the likelihood that the RETE match has compiled the four joins in the optimal order? The compilation is done by the lexical order of the condition terms. Therefore it is fairly likely that the optimal order is not used. Since TREAT will optimize the order of the joins on every cycle, it is a fair assumption that despite the lack of beta-memories, the number of parallel match operations performed by TREAT will be, on average, the same as RETE. We conclude that the beta-memories do not reduce the average number of parallel matches; therefore, it is not worthwhile to expend the time and space required to construct and maintain the beta-memories.
6.1 Performance Estimates for TREAT on a Medium Grain DADO

The parallel implementation of RETE includes the parallel associative look up of conflict set elements to be removed when a WM-element is deleted. With the exception of the construction of the beta-memories, the activities of the two algorithms are almost identical. Gupta estimates for RETE that the average cost of adding a WM-element is 3750 instructions. Of these, 940 instruction are needed to construct the beta-memories. A detailed explanation of this may be found in (Miranker, 1984). The estimate for the cost of deleting a WM-element is 1800 instructions. Of these, 330 instructions are required to process the beta-memories. On average, there are 2.5 changes in WM per cycle. Rule selection and right hand side evaluation is assumed to take 500 instructions. The total number of instructions per cycle is then:

\[
1.25 \times ((3750-940) + (1800-330)) + 500
\]

\[
= 5850\text{ instructions.}
\]

These instruction counts are based on a DADO PE constructed out of an 8 bit I address processor running at a speed of 2msec per instruction. Where Gupta has predicted performance for the DADO 2 prototype using the RETE match to be 67 production cycles per second, the TREAT algorithm is capable of 85 production cycles per second. Furthermore, it has been noted that the size of the beta-memories is often explosive (Gupta, 1984b), containing the full cartesian product of the antecedent memories. Thus, TREAT is more space efficient as well.

Similar arguments modifying the original DADO algorithm for a fine grain DADO have been able to show an improvement from 11 production cycles per second to 192 production cycles per second. Space does not permit a complete analysis here. The reader is encouraged to see (Miranker, 1984) for details.

7 CONCLUSIONS

The TREAT algorithm overcomes disadvantages of the original DADO algorithm by saving state across production system cycles. However, the internal structures of TREAT are simpler than those of the RETE match. As a result, TREAT may dynamically optimize the order of match operations on the WM and thus efficiently execute both temporally redundant and nontemporally redundant production systems.

Using the expanded abilities of TREAT and DADO, a new, more powerful production system language, HerbAI, is being designed to capture more parallelism than is possible to express in OPS.

We note that Gupta reports that a VAX-780 running the fastest OPS interpreter to date, OPS5 (Forgy83, 1983), is capable of only 30 to 50 production cycles per second (Gupta, 1984a). A DADO machine, using similar processor technology, is expected to perform 85 production cycles per second on OPS5 style systems. Yet such a DADO machine is considerably simpler and less expensive than a VAX-780.

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